Sección Especial: Óptica No Lineal / Special Section: Non-linear Optics

Nonlinear waves in optics and ultracold atomic gases

Ondas lo lineales en óptica y en gases de átomos ultrafríos

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ABSTRACT:

In this paper we analyse various applications of nonlinear waves governed by nonlinear Schrödinger equations. We look at three types of systems: ultracold atomic gases (Bose-Einstein condensates) periodic photonic structures and optical media with high-order nonlinearities. Results are presented as relevant applications ranging from the design of atomic lasers to new quantum phase transitions in optical systems.

Keywords: Nonlinear Waves, Solitons, Bose-Einstein Condensates.

RESUMEN:

En este trabajo se analizan diversas aplicaciones de ondas no lineales gobernados por ecuaciones de Schrödinger no lineales. En concreto, estudiamos tres tipos de sistemas: gases de átomos ultrafríos (condensados de Bose-Einstein) estructuras fotónicas periódicas y medios de ópticos con no linealidades de alto orden. Se presentan como resultados más relevantes aplicaciones que van desde el diseño de láseres de átomos a nuevas transiciones de fase cuánticas en sistemas ópticos.

Palabras clave: Ondas No lineales, Solitones, Condesados de Bose-Einstein.

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1. Nonlinear waves in ultracold atomic gases

In our group we study new devices that allow to extract matter-wave solitons[1] from a Bose-Einstein condensate reservoir in a controlled way and techniques for controlling their motion. The common idea in the different works is to induce a spatial variation of the scattering length along the longitudinal axis of the condensate from positive (or zero) to negative values. This can be done by magnetic [2] or optical [3] means.

In [4] we have designed a pulsed atom laser. When the zone in which the scattering length is switched to negative values overlaps the wing of the cloud, the condensate feels a repulsive force, which depends on the number of atoms in the BEC. If the change in the potential is large enough, the trap is overcome and part of the cloud is delivered and emitted outwards. When the condensate refills the gap left out by the outgoing pulse the process starts again and a new soliton would be emitted. This process would continue while there is a large enough remnant of atoms in the trap and would lead to a soliton burst escaping from the BEC.

In Fig. 1 it is shown the variation in the emission for different shapes of the region with negative scattering length created by an optical laser. For laser beams of finite spatial width, once the emitted solitons reach the opposite edge of the laser, the soliton is reflected back to the BEC and thus can remain trapped by the Gaussian beam in the vicinity of the BEC reservoir (see Fig. 1(c)), This effect allows the control of the emitted solitons once outcoupled and in the design of practical devices like laser tweezers for atoms [5]. In [6] we describe a simple technique that can be used to accelerate an ultracold beam of neutral atoms stored in a ring reservoir [7] by using an amplitude-modulated optical potential. The spreading of the atomic cloud is avoided by an adequate tuning of the scattering length.

The first step towards the experimental realization of the previous theoretical proposals is the creation of a Bose-Einstein condensate. In our group we are designing and building an experimental setup for getting a BEC with Rubidium (^{87}Rb) atoms.

In Fig. 2 we can see the vacuum chamber as it is in the present. The design of the vacuum system is based in a two-cell set up. In the first cell we create a magneto-optical trap (MOT) [9] for cooling atoms to the μK regime. These precooled atoms are then sent to the second cell (the scientific cell) for the final cooling stages (including evaporative cooling) needed to reach the condensation state. The first cell is fed with rubidium from an Rb getter. A differential vacuum system is created by separating the cells with a tube 8 cm long and 4 mm wide and pumping each section separately. The atoms are pushed towards the scientific cell by using a laser beam focalized in the center of the MOT creating an atom faucet [10]. The magnetic field in this cell is created by a quadruple-Ioffe configuration trap[11].

The optical setup uses two tunable lasers locked by rubidium polarization spectroscopy [12]. One of the lasers is employed to cool the atoms by using the Doppler effect and the other repumps the atoms to the correct frequency level. The slightly different frequencies needed during the various experiment stages are obtained by the use of acousto-optical modulators (AOMs).

The whole experiment is controlled by a computer program in C that sets the frequencies of the lasers, the switching times of lasers and magnetic fields, and controls the imaging system.

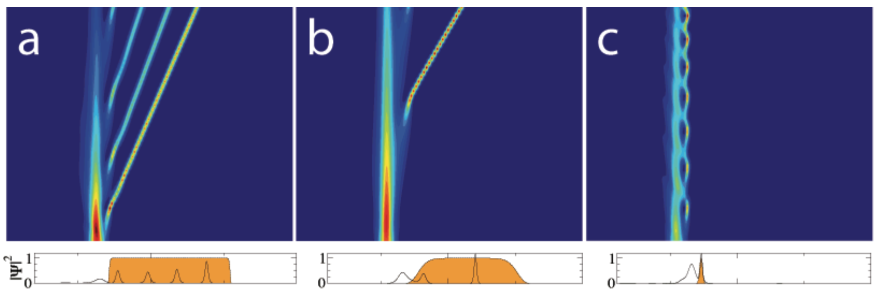

Fig. 1. Emission of atomic solitons from a BEC reservoir for different shapes of the laser beam corresponding with the shaded region under the dashed lines in the plots below each numerical simulation. Vertical axis is time.

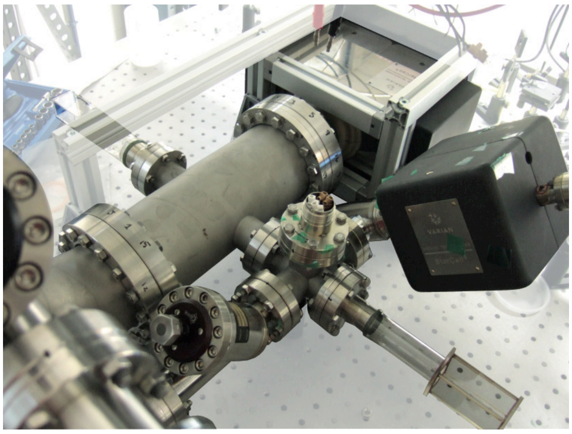

Fig. 2: Vacuum chamber for creating a BEC of ^{87}Rb atoms.

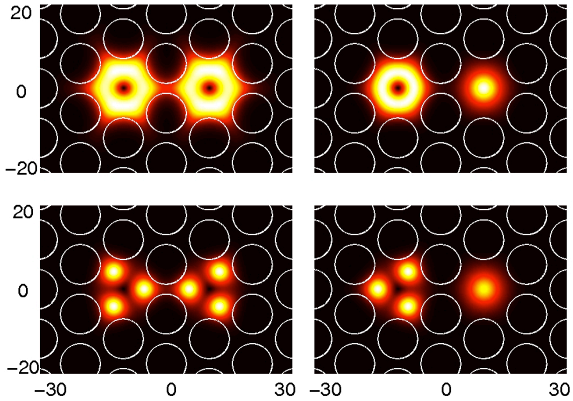

Fig. 3: Different types of nonlinear vortex-states in a dual-core PCF with Kerr nonlinearity.

2. Photonic periodic structures

Different research lines on nonlinear photonics are being undertaken at the moment, namely studies of solitons in periodic nonlinear lattices and in nonlinear plasmonic planar structures. On the other hand, we are also starting experimental work on photonic crystal fabrication for optical and THz frequencies.

Concerning solitons in nonlinear lattices we have studied patterns of multi-hump solitons that can exist in laser-induced periodic patterns on photorefractive media [13]. These systems are interesting because of the possibility of reconfiguring them in real time, leading to a higher flexibility for the control and manipulation of light properties.

On the other hand, we have also studied different kinds of solitons in nonlinear photonic crystal fibers (PCF) with a solid defect-core. Particularly, it was shown the existence of two-component (vectorial) fundamental solutions in a single core [14] incoherently coupled through the nonlinear Kerr interaction. These stationary solutions are stable in a range of powers and coupling coefficients. Besides, two-core systems made of two close defects forming directional couplers were also studied and their switching properties analysed [15]. The main result is the existence of an asymmetric mode that bifurcates from the symmetric one over a power threshold, suppressing the power transfer to the second core and leading to the power switching. Finally, those nonlinear directional couplers were studied considering different vortex-states [16], resulting in states, which can break the symmetry of the network for high enough powers due to the nonlinearity (see Fig. 3).

The recent activities in the nonlinear plasmonics line concerned the study of slot waveguides formed by the combination of metallic and nonlinear dielectric layers [17]. The inclusion of metals in such kind of devices allows to confine the field in sizes much smaller than the wavelength, typically tenths of nanometers for visible light. This leads to the possibility of overcome the diffraction limits and make devices compatible with nowadays electronics. On the other hand, plasmonic devices combined with periodic structures may be used to increase the efficiency of solar cells, which is another topic of interest at the moment. Particularly, we studied the switching operations in a directional coupler made of nonlinear dielectric cores embedded in metallic claddings [17]. The device was modelled performing Finite Difference in Time Domain (FDTD) simulations (see Fig. 4). Currently we are improving the numerical FDTD code to simulate more complicated devices and also to simulate periodic structures.

Finally, we are also starting experimental activities aimed to the fabrication periodic structures (photonic crystals (PC)) by means of two different techniques. On one hand we intend to use the technique of holographic lithography, making different plane waves interfere to create the periodic pattern. For this purpose we have the collaboration of researchers from the Instituto de Cerámica at the Universidade de Santiago, who are designing and fabricating special photosensitive materials, which allow to register the pattern in the photoresist. This pattern can be later turned into the final PC by processing the substrate at high temperature. This avoids the necessity of further steps of infiltration with a new material, avoiding imperfections derived from this additional process. Besides, introducing defects by laser irradiation is a simple task. The second technique we are developing is Direct Ink Writing (DIW), which allows depositing a material in a semi-fluid state on the substrate by pushing it through a micro-Syringe. A later thermal processing will turn it into a harder consistent state. This technique allows the fabrication of woodpile-like photonic crystals and simplifies the introduction of defects in order to make operative devices.

Another line, directly related to the latter, is the design and fabrication of devices to operate at frequencies in the THz band. A lot of attention has been put recently in these frequencies due to the new and interesting applications found, like tomography for evaluation of life tissues or detection of a big number of chemicals like drugs or explosives. Our aim is to fabricate photonic crystals for this band in order to control and guide the radiation and make operative devices.

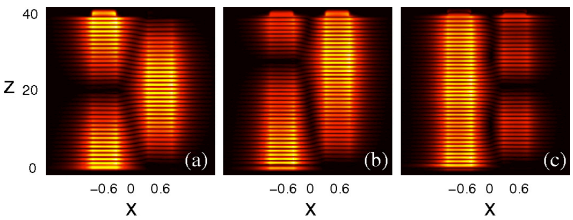

Fig. 4: FDTD simulation of the switching process in a two-slot directional coupler made of two nonlinear cores and metallic claddings. (a) Low power (linear) regime; (b) intermediate power; (c) large power regime (nonlinear switching).

3. Higher-order interactions

3.1 Optical filaments

Nowadays, both theoretical and experimental studies on nonlinear beam filamentation of ultrashort pulses are being actively developed [18]. When propagating through optical media with Kerr focusing nonlinearity, ultrashort laser pulses with power exceeding a certain threshold may become unstable and break up into several uncorrelated light structures by modulational instability (MI) processes. These optical structures, called *filaments* in this context, are spatiotemporal soliton-like light distributions that arise after MI and hold for large distances without changing their shape and size [19], due to a balance between the focusing Kerr nonlinearity and plasma-induced defocusing.

In particular, the dynamics of filament formation and evolution in carbon-disulfide cells has been recently analysed both experimentally and by numerical means, revealing that filaments can exist in such media below the threshold for the existence of non-negligible plasma effects [20]. This situation has been succesfully modeled assuming a Cubic-Quintic (CQ) optical response for the medium [21].

In a recent paper [22], we have theoretically described the results obtained in [20] by modelling the paraxial propagation of a high-intensity laser pulse through a CS_2 bulk, within the framework of the nonlinear Schrödinger equation (NLSE). In order to simplify the theoretical model, we assume a scalar slowly varying spatial envelope and we neglect group velocity dispersion effects. We also include a cubic-quintic nonlinearity, resulting in the following nondimensional NLSE

$$i\frac{\partial\Phi}{\partial\eta}+\Delta\Phi+(|\Phi|^2-|\Phi|^4)\Phi=0 \qquad (1)$$

where η is the propagation distance, $\Delta=\partial^2/\partial\chi^2+\partial^2/\partial\zeta^2$ is the 2D-transverse Laplacian operator and is the complex slowly varying electric field envelope propagating through the nonlinear medium. Eq.(1) admits plane-wave (PW) solutions of the type $\Phi=\Phi_0 e^{i\gamma\eta}$, where Φ_0 is a constant depending on γ eigenvalues as $|\Phi_0|^2=1/2+\sqrt{1-4\gamma}/2$. We have given a simple theoretical explanation of the results obtained in [20] by means of a linear stability analysis of plane waves [22]. After some algebra, we obtained the following analytical relation for the maximum growth rate of the perturbational modes $\Gamma_{max}=\Phi_0^2-2\Phi_0^4$, which indicates that the PW with $|\Phi_0^2|<0.5$, are modulationally unstable [22]. Another striking feature of this system is that the former expression for Γ_{max} predicts a cutoff for the MI. This modulational instability suppression occurs whenever $|\Phi_0^2|>0.5$, because in such case Γ_{max} becomes negative and the perturbational modes are not allowed to grow up. Notice that this cut-off does not exist in pure Kerr media, in which all the plane waves are unstable.

Liquid light condensates are optical solitons in CQ optical media with a "flat-top" transverse spatial envelope and intriguing surface tension properties[23]. Recently, it was shown that their dynamics follows the same equations governing the evolution of usual liquid droplets [24]. We have shown a mechanism for the dynamical excitation of these structures by means of spatial control of the filamentation regime and coalescence processes between self-guided optical channels[22]. This process is sketched in Fig. 5.

We have considered an initial condition for our simulations consisting on a broad-extended laser pulse with random noise fluctuations added. On top of this light distribution we have induced a small scale Gaussian fluctuation (SGF) in order to select a small region where filamentation will first appear (see Fig. 5(a)). Once the pulse has entered the medium, the *SGF* starts to self-focus. It is remarkable how such a process is fastly counteracted by the self-defocusing nonlinearity. As a result, a bright hot spot is observed at the distribution centroid and diffractive rings representing shockwaves due to "background-fluctuation" interplay appear. These rings lead to the formation of new filaments from pulse MI (see Fig. 5(b)). The arising filaments are then used to provide an energy source for the central seed so that coalescence processes could increase its power, if the phases of the soliton and the filament match.

As shown in Fig. 5(c), there is indeed a dynamical energy exchange between filaments and hot spot during propagation. As a result, all solitonic structures are both created and annihilated several times, until a wider structure with almost constant peak intensity arises (see Fig. 5(d)). As a consequence, the emerging liquid light soliton [23] is strongly perturbated by the energy excess released in the collective coalescence. Nevertheless, from Fig. 5(e) we see that after a large propagation distance the flat-topped soliton features an improved radial symmetry, the energy excess being radiated in the form of linear modes. In the last snapshot Fig. 5(f) we show the phase map of the field depicted in Fig. 5(e). From this phase structure, it can be stated that the light condensate appearing at the last stage of the propagation is a spatially coherent light structure since the colour within the region where it is located (see blue circle in Fig. 5(f)) is homogeneous.

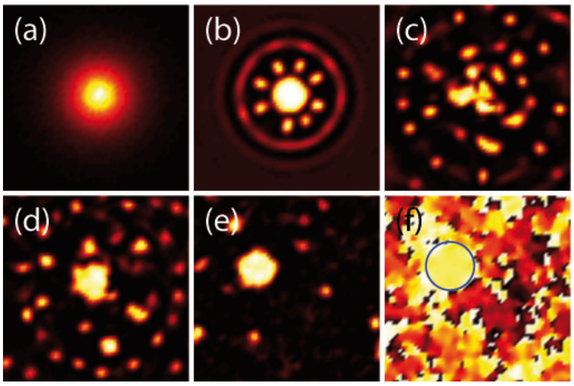

Fig. 5: Intensity pseudocolor plots of the SGF evolution in the presence of an unstable homogeneous back- ground. The spatial domain is [-75 < χ, ζ < 75] and propagation are: (a) η=0; (b) η=80; (c) η=150; (d) η=200; (e) η=1000. Last picture (f) displays the phase map of the field depicted in (e).

3.2 Photon-Photon scattering in vacuum

Under usual conditions, light propagates in vacuum according to the linear Maxwell equations, so that different electromagnetic waves do not keep memory of their possible crossing in the way to their reception points. However, this superposition principle is expected to be violated due to photon-photon scattering in vacuum (PPSV) as predicted by Quantum Electrodynamics (QED) and non-standard models such as Born Infeld theory or in new physics scenarios involving minicharged or axion-like particles [25]. In Ref. [26], we have shown that this effect implies that two crossing waves will phase shift each other. The computation of the resulting phase shifts for two counter-propagating laser pulses of parallel and orthogonal polarizations were computed in Ref. [27] and [25], respectively. These effects were argued to be measurable at the QED rate in future exawatt facilities such as ELI [28], and can be used to improve the PVLAS [29] limits on PPSV at present facilities.

More recently, we have proposed a more sensitive technique to measure the phase shift due to PPSV using optical measurements [30]. In fact, we have shown that due to PPSV a laser pulse can be diffracted by a more concentrated high power pulse even in the absence of matter. In our experimental design a polarized ultrahigh power Gaussian pulse A of transverse width ω_A

crosses an almost contra-propagating polarized 'probe' laser pulse B of width $\omega_B \gg \omega_A$. For simplicity, we assume that the two beams have the same mean wavelength $\lambda = 2\pi/k$ and frequency $\nu = c/\lambda = ck/2\pi$, although in principle they may have different durations τ_A and τ_B.

From Ref. [25], we learn that the central part of the probe B, after crossing the pulse A, acquires a phase shift $\phi_{L,T}(0) = I_A(0) k \tau_A a_{L,T} \xi_{L,T}$, where $I_A(0)$ is the peak intensity of the high power beam at the crossing point, the indexes L and T refer to the two beams having parallel or orthogonal linear polarizations, respectively, and we have defined $a_L = 4$ and $a_T = 7$. Here, ξ_L and ξ_T are the paramenters that describe PPSV at optical frequencies as described in Ref. [25].

The QED prediction is $\xi_L^{QED} = \xi_T^{QED} \equiv \xi = 8\alpha^2\hbar^3/45m^4c^5 = 6.7 \times 10^{-30} m^3/J$. The inner radius r_0 of the ring detector S is obtained by requiring that the total power due to the diffracted wave for $r > r_0$ is much larger than the power of the non-diffracted wave in the same region, say by a factor 100. The number of diffracted photons that will be detected after $\mathcal{N}$ repetitions of the experiment is then

$$\mathcal{N}_D^{\mathcal{N}} = \frac{8f\mathcal{N}}{\pi\hbar c} \frac{E_A^2 E_B \omega_0^2}{\lambda \omega_A^4 \omega_B^2} \left(e^{-\frac{2r_0^2}{\omega_D^2}} - e^{-\frac{2R^2}{\omega_D^2}} \right) (a\xi)_{L,T}^2, \quad (2)$$

where f is the efficiency of the detector, R is the external radius of the detector, and $E_A = P_A \tau_A$ and $E_B = P_B \tau_B$ are the total energies of the two pulses. In Ref. [30], we have shown how to optimize this number of scattered photons taking into account realistic angular constraints for the experiment. For simplicity, we suppose that both pulses A and B can be obtained by dividing a unique high power pulse. Using e.g. a 100 Petawatt laser such as ELI [28], and assuming a duration $\tau = 30\,fs$, energy $E = 3\,kJ$ and wavelength $\lambda = 800\,nm$, we obtain that our proposed experiments can resolve ξ_L and ξ_T as small as $\xi_L^{limit} = 2.8 \times 10^{-30} \mathcal{N}^{-1/2} m^3/J$ and $\xi_T^{limit} = 21.6 \times 10^{-30} \mathcal{N}^{-1/2} m^3/J$, values that are well below the QED prediction even for a single-shot experiment (N = 1). As a result, our proposal constitutes the most sensitive framework for the detection of PPSV using only optical measurements.

