Sección Especial en Óptica Cuántica / Special Section on Quantum Optics

Quantum physics with multimode light, electronic waveguides and driven oscillators at IFISC

Física cuántica con luz multimodo, guias de ondas electrónicas y osciladores forzados en el IFISC

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ABSTRACT:

We present here a brief overview of the main topics studied in the Institute for Cross Disciplinary Physics and Complex Systems (IFISC) in the line of quantum optics and information, and nanoscience. Our work ranges from the properties of light, such as the formation of optical patterns in nonlinear media and the uncertainty properties of angular momentum vs. angle, to the properties of electronic waveguides where propagation and interference effects can be investigated with great detail. We also mention some recent activity on the properties of quantum to classical transition for oscillators, the sustainability at high temperatures of their entanglement and how to probe a phase transition in a spin chain through the change of the state of the emitted light.

Keywords: Multimode Quantum Optics, Optical Parametric Oscillators, Optical Angular Momentum, Electronic Wave-Guides, Entanglement, Quantum Phase Transitions.

RESUMEN:

Presentamos aquí un breve resumen de los temas estudiados en el Instituto de Física Interdisciplinar y Sistemas Complejos (IFISC) en las líneas de óptica e información cuánticas y nanociencia. Nuestro trabajo abarca desde las propiedades de la luz, tales como la formación de patrones en medios no lineales y la incertidumbre ángulo-momento angular, hasta las propiedades de guías de onda electrónicas donde la propagación e interferencia se pueden estudiar con gran detalle. También mencionamos actividad reciente sobre las propiedades de transición del comportamiento cuántico a clásico para osciladores, la sostenibilidad a altas temperaturas de su entrelazamiento y cómo detectar una transición de fase en una cadena de espines a través del cambio de estado de luz emitida.

Palabras clave: Óptica Cuántica de Muchos Modos, Osciladores Ópticos Paramétricos, Momento Angular Óptico, Guías de Ondas Electrónicas, Entrelazamiento, Transiciones de Fase Cuánticas.

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1. Introduction

One of the research lines at the Institute of Cross Disciplinary Physics and Complex Systems (IFISC) focuses on the theoretical study of specific topics in quantum optics, quantum information and nanoscience [1]. Charge and spin transport (nano-electronics and spintronics) are studied in semiconductor nanostructures, including quantum dots and wires. The possibility to control photonic properties, such as quantum correlations and entanglement in light beams, are studied in nonlinear optical devices, cold atoms and lasers. General properties shared by these systems are studied in the context of quantum information, focusing on the identification of entangled states, the characterization of their degree of entanglement and its creation and dynamical evolution. In this article we briefly review some research topics such as quantum aspects of the formation of optical patterns in nonlinear media, the uncertainty properties of angular momentum vs. angle, and the properties of electronic waveguides where propagation and interference effects can be investigated with great detail. The quantum to classical transition for oscillators and the sustainability at high temperatures of their entanglement is also part of our research. We finally mention how to probe a phase transition in a spin chain through the change of light emission from a Fock to a coherent state.

2. Quantum aspects of spontaneous patterns formation

Light-matter interaction is at the heart of both non-linear and quantum optics, leading to rich temporal and spatial dynamical effects and allowing for the generation of non-classical states of light. In particular, light interacting with non-linear media in optical cavities gives rise to the phenomenon of spontaneous pattern formation in which the output light shows, for instance, a periodic structure (stripes, squares, hexagons...) or localized spots (like solitons), breaking the transverse translational symmetry. Two decades ago non-classical spatial phenomena where predicted in such self-organization processes, extending observations from the purely temporal (one mode) to the spatio-temporal domain (multimode quantum optics) [2].

Initially, quantum effects as spatial entanglement and multimode squeezing have been predicted in 'quantum images', below the threshold of pattern formation [2]. By considering fully multimode models, we predicted non-classical correlations also above the threshold for pattern formation, in presence of intense patterns, both in stable regular [3] and disordered structures (frozen chaos) [4]. Other regimes, like the convective one, in which drifting patterns are sustained by quantum noise, confirmed the importance of the stability of the spontaneous structure in order to obtain spatial quantum effects [5]. More recently we have been considering the possibility to engineer spatial quantum fluctuations of light emitted by an optical parametric oscillator when a photonic crystal is taken into account transversally. The effect is a modulation of the refractive index in the cavity allowing to tune the noise intensity in the output signal (below threshold) and also to improve spatial squeezing and entanglement (above threshold) [6]. Different approaches have been used to study within the quantum formalism such nonlinear and multimode optical devices, mainly based on quasi-probabilities formalism and numerical simulation of stochastic equations [7,5,4].

3. Optical angular momentum

Light is known to carry not only spin angular momentum, related to circular polarization, but also orbital one, associated to the beam spatial structure [8]. The definition of angular momentum of light, $\mathbf{L}=\int d\mathbf{x}\ \mathbf{r}\times\mathbf{p}$, comes naturally from the linear momentum $\mathbf{p}$, being the latter proportional to the Poynting vector. A singular beam with azimuthal phase dependence $\exp(i\ell\phi)$ has an associated orbital angular momentum in the propagation direction of $\ell\hbar$

photon, as evident from the rotation of particles trapped in such beams [9]. Any multimode paraxial beam $\psi(r,\phi,z)$ propagating in the z direction can be decomposed in its spiral components as $\psi(r,\phi,z)=\gamma\sum_\ell a_\ell(r,z)\exp(i\ell\phi)$ (with γ constant) with $P_\ell(z)=\int|a_\ell(r,z)|^2 r dr$ [10,11]. Then the average orbital angular momentum is $L_z \propto \sum_\ell P_\ell \ell = \overline{\ell}$.

An interesting question is how to measure such spatial spectrum P_ℓ of a generic light beam. If we are interested in the Fourier spectrum it is well known that a simple lens provides the answer while in Ref. [12] it was proposed an efficient method allowing the experimental determination of the spiral spectrum P_ℓ. The generalization to measure the entire spatial mode spectrum, for instance in Hermite-Gaussian components, of any arbitrary monochromatic paraxial beam was proposed in Ref. [13]. Both proposed set-ups consist of interferometers and optical elements such as Dove prisms or a combination of spherical and cylindrical lenses.

Measuring with the interferometer proposed in Ref. [12] the spiral spectrum of a paraxial light beam in a Laguerre-Gaussian mode $\psi(r,\phi,z)$ = $\gamma a_\ell(r,z)\exp(i\ell'\phi)$ and aligning the set-up with the propagation axis, only one component $P_{\ell'}$ would be not vanishing. An important observations is that, in spite of the generally invoked intrinsic character of the optical angular momentum, only its average is independent on the alignment, while a broadening generally appears when changing the position of the axis about which the angular spectrum is retrieved; orbital angular momentum of light can therefore be denoted as "quasi-intrinsic" [12]. As a matter of fact, the measured spectrum of a Laguerre-Gaussian mode with respect to a shifted axis would contain an infinite number of components.

Considering that a Laguerre-Gaussian mode has a well defined spiral component (about the propagation axis) and therefore a vanishing orbital angular momentum variance ($\Delta L_z = 0$), an interesting point is about the variance in the conjugate variable, that is the angle ϕ. The uncertainty relation between angular momentum and angular position is more complicated than for linear momentum and position ($\Delta x\, \Delta p_x \geq ½$) being

$$\Delta\phi\,\Delta L_z \geq \tfrac{1}{2}\left|1-2\pi P(\phi=\pi)\right|. \tag{1}$$

Indeed, unlike the linear position, the angle takes values only over a finite range, here $[-\pi,+\pi]$, so that $\Delta\phi$ must have an upper bound for physical states. This is at the origin of difference between the uncertainty relation for angular and linear variables. It has been by means of an optical implementation that the *angular* uncertainty relation (1) was actually measured for the first time [2] using light beams in Gaussian modes and passing them through proper apertures. The states that satisfy the equality in an uncertainty relation are sometimes referred to as intelligent states and in the case of position and momentum have Gaussian probability distributions. These intelligent states have also the property to minimize the uncertainty product for a given position uncertainty given the momentum one. For the corresponding angular uncertainty relation (1), however, intelligent states need not be minimum uncertainty product states. Of course there is not violation of the Heisenberg relation but we can say that there are different ways to define a minimum uncertainty state. One is looking at a global minimum (here any Laguerre-Gaussian mode is an angular momentum eigenstate and gives this minimum $\Delta\phi\Delta L_z=0$), or at (intelligent) states that fulfill the equality in the uncertainty relation (1) or considering other constraints, for instance states with a given uncertainty in angular position and minimizing the uncertainty product. These "constrained minimum uncertainty product states" have been calculated and compared with intelligent states in Refs. [14-16].

As an example of an application of spiral beams carrying orbital angular momentum we can mention the possibility to increase resolution in rotation measurements [17]. It is well-known that the limiting resolution in optical interferometry is set by the number of photons used, with the functional dependence determined by the state of light that is prepared. When measuring the rotation of a beam of light about an optical axis, we showed that is also important the choice of the spatial profile of the used light beam: The limiting resolution depends indeed on the total number of quanta of orbital angular momentum carried by the light beam [17].

4. Spin-orbit coupling in electronic waveguides

Analogies between optics and electronics arise mainly because electrons behave as waves when the size of the electronic device is reduced down to the nanometer scale. In today's solid-state nanostructures, samples are relatively free from unwanted impurities or defects and the electronic dynamics can be then considered as ballistic. Since the electronic wave function conserves its phase during transport, wave superposition and interference effects can be observed in experiments.

Typically, electrons are confined to two-dimensional layers formed at the interface between two semiconductor compounds with different gap (e.g., GaAs/AlGaAs). In the presence of an additional confinement potential, one can construct and investigate narrow channels which permit the motion of traveling electron waves much like optical waveguides are designed by spatially varying the dielectric constant. The states in an electronic waveguide consist of transversal modes with an energy spacing which depends on the characteristics of the confinement potential and an energy dispersion which is quadratic in the wavevector (and not linear as with photons). A measurement of the linear conductance of an ideal waveguide yields quantized values at $2ne^2/h$ (n=1,2…). We note that this relation depends on physical constants only and is insensitive, to a great extent, to the details of the electron spectrum.

On the other hand, there has recently been considerable effort in controlling and detecting the electronic spin state in nanodevices. The manipulation can be done by means of external gates which locally modulate the strength of spin-orbit interactions. Such coupling between spin and orbital angular momenta has been also observed in polarized light but the effect seems to be much weaker. On the contrary, in semiconductor heterostructures the spin-splitting due to spin-orbit interactions is of the same order as typical energy scales of the system such as the Fermi energy (a few meV). In electronic waveguides, the interaction takes place between the spin **s** and the linear momentum **p**, namely, $H_{so} = \alpha(\mathbf{s}\times\mathbf{p})\cdot z$, where α is the spin-orbit strength and z is the direction perpendicular to the waveguide plane. The Hamiltonian term H_{so} can affect strongly the energy dispersion, which presents minima and maxima when an external magnetic field is applied parallel to the waveguide. As a consequence, the conductance curves show anomalous steps as a function of the Fermi energy, although these can disappear if spin-orbit intermode coupling terms are included in the calculation [18]. In fact, we have recently demonstrated that interaction between adjacent energy modes due to spin-orbit coupling is crucial to correctly describe conductance modulation and polarization effects in spin-based field-effect transistors [19].

When the spin-orbit coupling acts only around a finite region of the waveguide, an interesting phenomenon occurs [20]. The spin-orbit inhomogeneity acts effectively as an attractive impurity that traps electrons in localized quantum levels (bound states). As a result, scattering off the impurity greatly reduces the waveguide transmission from the ideal value $2ne^2/h$. Furthermore, we find that the spin-orbit interaction is also able to provide a coupling channel that connects the localized levels to the propagating states, the bound state thus acquiring a finite lifetime. Therefore, the direct transmission channel interacts with the hopping path through the quantum level and the resulting path interference gives rise to characteristic asymmetric line shapes known as Fano resonances [21]. The formation of the bound state arises from the coupling between traveling waves at a given Fermi energy and evanescent modes with energies above the Fermi energy. The role of evanescent modes in electronic waveguides with spin-orbit interaction has been recently emphasized [22]. The Fano resonances can also occur in purely one-dimensional waveguides provided an external magnetic field is applied along the waveguide axis [23].

In contrast to photons, electrons repeal each other due to Coulomb interaction. Electron-electron interactions are particularly strong in low dimensional systems since confinement potentials weaken electronic screening. In spin-orbit waveguides, interaction effects are expected to take place at energies around the Fano resonance. In fact, we find charging energies higher than the level broadening [23]. Using an Anderson model, we predict the formation of local magnetic moments and finite spin-polarizations even out of equilibrium, when an electric bias is applied [24]. At low temperatures, electronic transport is dominated by Coulomb blockade resonances as the spin-orbit strength is tuned at a fixed value of the Fermi energy [23]. If temperature is further reduced, there arises an effective antiferromagnetic interaction between the quantum impurity and the conduction electrons. This is the celebrated Kondo effect, which manifests itself as a narrow resonance at the Fermi energy. We have recently investigated how the spin-orbit interaction splits the Kondo resonance [25]. The complete transmission limit can be achieved only if a compensating in-plane magnetic field is applied to the system.

In conclusion, electronic waveguides are an interesting area of research where propagation and interference effects can be investigated with great detail. We find that the transmission properties of the system are affected in a dramatic way when the electronic spin and the orbital degrees of freedom are coupled via an externally controlled interaction. We expect that further analogies between optics and nanoelectronics will be especially rewarding in this topic.

5. Quantum to classical transition in continuous variable systems

The transition from classical to quantum behaviour in systems with continuous variables has a long tradition both in the field of quantum statistical physics as well as in quantum optics and quantum information. It is typically assumed that interaction with the hot environment leads to fast loss of quantum coherence and any other indicator of quantum properties such as entanglement. Hence, the quantum regime is understood, when open dynamics is considered, as limited by the temperature of the heat reservoir around the system.

Though this has been the subject of much research, recently we discovered that nonequilibrium states exist in which a finite temperature is not an impediment for creation and preservation of entanglement between two harmonic oscillators which are linearly coupled and subjected to parametric driving [26]. If the experimentalist is able to engineer a high enough coupling together with driving of the frequency or coupling, the temperature at which entanglement can be observed can raise by several orders of magnitude; in the case of oscillators in the 20GHz frequency range, this would account for observation of entanglement at room temperature. It was previously demonstrated [27] that attempts to preserve entanglement in static situations for open systems are only possible if we are to assume rather stringent conditions, namely: identical frequencies and a common reservoir for both oscillators. Any other case will yield a finite time for entanglement sudden death. Thus, quantum features at high temperatures seems to be plausible only for nonequilibrium processes. The parametric driving can of course be a simple sinusoidal function, but can be optimized even in the presence of many more coupled oscillators [28] to yield the maximum possible entanglement for a given maximum experimentally available driving amplitude and time. Needless to say that any such protocol has a given energetic cost. Such cost can be calculated (and optimized) for the case of a parametrically driven harmonic oscillator as the irreversible thermodynamic work dissipated by the system [29]. This work scales exponentially with the amount of squeezing (and thus entanglement, if applied correctly), meaning that it is rather costly to produce entanglement through parametric driving.

Finally, we proposed the exploration of these nonequilibrium high temperature entangled states [26] in e.g. planar Penning traps [30], due to the additional flexibility they provide in order to achieve a stronger coupling, as compared to standard Penning traps. In addition, a modulation of the frequencies is easily achieved with standard techniques.

6. Optical probe of a magnetic quantum phase transition

The interaction between light and matter can give rise to interesting phenomena whenever the coupling between them reaches some critical value. Here, we want to comment on a specific model [31] having the peculiar characteristic of allowing to detect the appearance of magnetic ordering by observing the transition from a Fock regime to coherent emission of the e. m. field.

To be more specific, let us consider an isotropic XY spin chain $J\sum_{\ell=1}^{N}[\sigma_{\ell}^{+}\sigma_{\ell+1}^{-}+\mathrm{H.c}]$, with nearestneighbor, and an electromagnetic mode a. Let them interact through the coupling between the displacement $a+a^{\dagger}$ of the e.m. field and the total spin polarization $\sum_{\ell=1}^{N}\sigma_{\ell}^{z}$. Calculating the ground state, one can observe an energy crossing that generates a sharp transition in both light and spins. The instability of the spin chain manifests itself through the appearance of a ferromagnetic (or antiferromagnetic) order, that would be forbidden by the symmetry of the Hamiltonian. The same symmetry would also forbid the coherent emission in the mode of radiation. The peculiarity of this model is that the two different symmetry-breaking mechanisms are tightly related and are driven by the change of the coupling constant g. The probing method can be experimentally implemented and simulated in arrays of trapped ions forced to interact using forcing laser fields. The coupling with the bosonic component arises from the vibrations of the optical traps induced by the forcing laser fields. Then, measuring these phonons through fluorescence emission techniques, a direct inspection of the internal ion states is obtained.

7. Conclusions

In this paper we have described some of the research topics carried out at IFISC having the common goal of understanding both fundamental *quantum* properties and potential applications of optical systems, like nonlinear cavities or singular beams, as well as systems analog to optical ones, as electronic waveguides, and also entangled harmonic oscillators, or hybrid systems like spins interacting with light.