Sección Especial en Óptica Cuántica / Special Section on Quantum Optics

Quantum integrated optics: Theory and applications

Óptica integrada cuántica: Teoría y aplicaciones

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ABSTRACT:

In this work we present a brief outlook of the main results of the research work developed by our group on quantum integrated optics. The major research of the group has been focused on integrated optics and besides in the last years several theoretical and applied quantum studies have been made about both linear and nonlinear integrated optical devices. These studies have had as a primary aim to accomplish a rigorous theoretical analysis of both the optical propagation and the characterization of quantum light states in waveguiding devices by means of the Momentum operator and the spatial Heisenberg's equations, and by using also quantum optical propagators calculated by the Feynman's formalism on the optical field-strength multimode space. In this space the probability distributions are well-behaved and moreover they can be measured by multimode homodyne and/or heterodyne detection techniques. Likewise some of the future researching lines on experimental and theoretical quantum integrated optics are indicated.

Keywords: Linear and Nonlinear Quantum Integrated Optics, Quantum Optical Propagation, Characterization of Quantum States.

RESUMEN:

En este trabajo se presenta una breve descripción de los principales resultados del trabajo de investigación desarrollado por nuestro grupo en óptica integrada cuántica. La principales líneas de investigación del grupo siempre han estado en la óptica integrada y en los últimos años se han desarrollado además varios estudios cuánticos teóricos y aplicados de dispositivos ópticos integrados lineales y no lineales. Estos estudios han tenido como principal objetivo el análisis teórico riguroso tanto de la propagación como de la caracterización de estados de luz cuántica en dispositivos de guía de onda mediante el operador Momento y las ecuaciones espaciales de Heisenberg, y también mediante propagadores opto-cuánticos calculados con el formalismo de Feynman sobre el espacio multimodo de campo óptico. En este espacio las distribuciones de probabilidad están bien definidas y además pueden medirse mediante técnicas de detección homodina y/o heterodina multimodo. Asimismo se indican algunas líneas de investigación futura tanto teórica como experimental sobre óptica integrada cuántica.

Palabras clave: Óptica Integrada Cuántica Lineal y No Lineal, Propagación Opto-Cuántica, Caracterización de Estados Cuánticos.

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1. Introduction

As it will be shown along this work the major challenge in quantum integrated optics has been to formulate a consistent quantum mechanical approach of the spatial optical propagation in integrated optical devices working with linearly and/or nonlinearly coupled 1D (2D) vectorial modes. These optical modes are spatially confined in two (one) dimensions and propagating in one (two) spatial dimension in integrated waveguides [1]. It is important to highlight that the integrated optical (or photonic) waveguides own as a common physical characteristic that the modal confinement extent is in the order of the light wavelength. The 1D and 2D modes are present in any conventional integrated optical waveguide which can be made in optical glasses, crystals, semiconductors, polymers and so on [1,2], or even in integrated guides made in less conventional materials such as photonic crystal waveguides and nano-photonic waveguides based on silicon or plasmonic materials. Likewise a wave-guiding structure is made up of several 1D and 2D integrated guides interconnected which forms an integrated optical device where the optical modes of the guides are usually coupled in a linear and/or a nonlinear way in order to implement a particular optical operation [1,2].

Since we are interested in propagation of quantum light we must underline first of all that a Hamiltonian description of the spatial quantum optical propagation in homogeneous optical media presents several conceptual and formal inconsistencies such as it was proved [3,4]. The above problem is still deeper when quantum optical propagation is studied in inhomogeneous optical media such as integrated optical waveguides [5]; indeed, the vectorial structure of the guided (confined) modes together with the intramodal, intermodal and chromatic dispersions, which are intrinsic to integrated optical guides, give rise to more serious inconsistencies and shortcomings when we use the Hamiltonian approach to describe spatial optical quantum propagation in integrated devices. Therefore in order to develop integrated optical devices operating with quantum light, that is, to both design and fabricate them, a consistent approach of the quantum propagation is required. This approach would must cover nearly all the kind of wave-guiding structures in such a way that a common quantum mechanical treatment can be used. Our group has acquired an enough experience in classical integrated optics [6] and, in a natural way during the last years, we have also focused on the problem of the quantum light propagation in integrated devices (theory) because it would allow us to carry out the both design and fabrication of quantum integrated devices in a more realistic way (applications). In fact it has been recently shown by Politi *et al* [7] that integrated optical devices can become a very serious solution to many quantum optical technological issues, and therefore a deeper quantum studies of integrated devices will be necessary.

The plan of this work is as follows: first of all, and for sake of consistency, we briefly present in section 2 the main characteristics of the Integrated Optics (IO); likewise we present some of the main results obtained by our group for the development of integrated optical components; we also highlight the recent experimental results in quantum integrated optics by underlining their extraordinary and undeniable impact on the present quantum technology. On the other hand, the quantum optical spatial propagation in homogeneous media was studied for first time by Abram [3] and Huttner *et al* [4], however this study in IO was made a few years ago [5], therefore in section 3 we will present the main results of our quantum propagation approach in IO by indicating *ab initio* how a general quantum Momentum operator can be derived; we also show that such an approach also allows to derive other quantum operators in IO; likewise, the propagation of probability amplitude distributions in an optical field strength multimode space is presented, which has an enormous interest because of its direct relation with homodyne and heterodyne quantum measurements. In section 4 we will concisely present the main results related to the optical characterization of quantum states and in section 5 a summary of the main aims for a future research work are commented.

2. IO and quantum optics

In this section we present both the concept of IO and the main integrated optical devices used to implement optical operations; it must be stressed that many of these devices have not a counterpart in bulk optics. Likewise we show that the theoretical and experimental results obtained in the last years confirm that a new researching field, which can be called *quantum integrated optics*, is emerging.

2.1 IO: concept and devices

IO concerns with the development of miniaturized optical devices of high functionality and its integration on a common substrate. Light sources, receivers, light deflectors, modulators, sensors are some examples of these devices. Devices are smaller, lighter, and more robust that their bulk counterparts because no alignment is needed. The most fundamental building block of these devices are the integrated waveguides. As it was commented above the transverse size of the waveguides is comparable to the wavelength of light so only one or a few guided modes (optical field distributions) propagate. The waveguides can be made by depositing material on top of a substrate and etching unwanted portions away; alternatively waveguides can be made by chemical or physical modification of the substrate in the region of interest. Therefore, the process of making waveguides requires a substrate and some micro-fabrication technologies to build them.

Glasses, dielectric crystals and semiconductors are used as substrate materials which give rise to different kind of devices: linear and non linear passive components (a component can be understood as a simple device), active devices (amplifiers and sources) and transducer devices (electrooptic, acoustooptic and magnetooptic ones) [1,2]. Our group has mainly developed linear and nonlinear passive components in glass by ion-exchange technology, thin film deposition and both standard and laser lithography [6]. We distinguish between slab components (based on 2D guides) and planar components (based on both 1D and 2D guides, that is channel guides and/or slab guides). When several of these components implement a specific operation then they constitute an integrated optical circuit [1,2]. Examples of these components are the linear and nonlinear slab waveguide lenses [8-10], slab mirrors [6,11], or gratings [1,2]. On the other hand, in optical planar components the confinement can also take place in two spatial dimensions by means of channel waveguides. The appropriate patterning of channel waveguides in a substrate allow us to build directional couplers, power distributors, splitters, multiplexers, ring filters, Mach-Zehnder interferometers, etc. We must underline that the integrated directional couplers play a significant role in IO [1] and together with integrated interferometers form the basis of many integrated optical devices for optical processing and/or sensing. An alternative to directional couplers is the Y junction or 1 to 2 power splitter. In the Y junction, a channel waveguide is split in two waveguides and if the two arms become N arms we obtain a 1 to N power splitter. In Fig. 1 is shown a 1×4 splitter; we must stress that 1 to N devices (linear and nonlinear ones) can be also achieved by means of slab components [10,12].

2.2 Glass IO: technology

Glass is one of the most used material in IO because of its low cost, excellent transparency, mechanical rigidity, high threshold to optical damage, and availability in both substantially large sizes and variety [2]. Moreover, glass substrates are amorphous, and it is easier to produce polarization-insensitive components in glass. In addition, the refractive index of glasses used in IO (e.g., silicate, phosphate) is close to that one of an optical fiber and, therefore, coupling loses between the waveguides made in glass and the optical fibers are smaller and therefore there is a high compatibility between glass integrated optical devices and optical fiber networks. It must be stressed that even amplifiers, lasers and frequency doubling have already been demonstrated in special glasses. A number of different processes have been employed to make glass waveguides which can be divided in: sputtering, chemical vapor deposition, sol gel coating, ion implantation and ion exchange. Ion exchange process has been by far the most popular technique to produce glass integrated optical components [2]. Our group has developed a wide set of ion-exchange techniques for optical integration in glass [6]. These techniques are being used at present to continue developing integrated optical devices.

On the other hand the complementary microfabrication technology is the lithography on glass. The purpose of lithography is to define the regions or patterns on the substrate where the components or circuits are going to be made. There are three primary exposure methods: contact, proximity, and projection. With these methods and using high intensity UV exposure systems we can get resolutions of about 1μm. Better resolutions are achieved with expensive projections systems using shorter UV wavelengths, however if sub-micron resolution is needed at a low cost, other techniques are wanted. This is the case for patterning very small circuits, as directional couplers with very sharp edges, at a standard laboratory. The most interesting technique which satisfies both requirements of resolution and cost for craft production is the laser beam lithography which has been widely developed in our group [13]. It allows us to get resolutions close to the most accurate techniques with an equipment affordable for most laboratories. As an illustrative example we show in Fig. 1 the result of a laser beam lithography corresponding to a 1 to 4 power splitter where the waveguide widths become lower than 1μm.

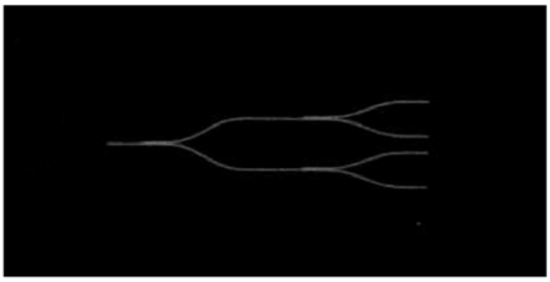

Fig. 1: Laser lithography of a 1x4 splitter

2.3 IO for quantum optics

In IO the first theoretical works on quantum propagation in integrated devices, such as a linear coupler, were made by Jansky *et al* [14] by means of a correspondence rule and therefore without a rigorous derivation of the Heisenberg's equations. Afterwards, by following a heuristic way in order to find the operator Momentum operator, studies for quantum linear couplers were presented by Toren and Ben Aryeh [15] and studies for nonlinear couplers [16] were presented starting from the seminal work by Perina [17]. Finally, a rigorous derivation of the quantum Momentum operator for all these integrated optical coupling devices operating under modal coupling was proposed [5]. All these studies defined a wide theoretical underground for quantum integrated optics, although there had been no particular quantum optical experimental application by using integrated components. However, two years ago Politi *et al* [7] provided the most shocking test of an integrated quantum optical device, that is, they showed in an experimental way how to implement a C-NOT gate by using integrated directional optical couplers and therefore confirming that IO can supply an important boost to quantum optics technology, by solving very up-to date problems related to quantum computation, quantum communications and so on. This preliminary result has opened a myriad of theoretical and experimental possibilities of researching and development, which can be gathered in a new topic that can be called *quantum integrated optics*. Obviously the IO is a well-established topic and it has produced a wide range of integration technologies and has furnished a wide variety of designs of integrated optical devices mainly intended for applications in optical processing, optical communications and optical sensing. However, starting from the Politi *et al*'s work the IO should be regarded, in our opinion, as a serious strategy to develop a powerful quantum optical technology. Obviously for that purpose a consistent theory of quantum optical propagation in IO is required and accordingly the new devices can become correctly designed for implementing quantum optical operations. During the last past years we have focused on both the development of a consistent theoretical approach to the spatial quantum optical propagation in IO and its characterization by combining theoretical tools come from both standard IO and standard quantum optics. The theoretical results obtained together with our experience in the development of integrated optical components is opening new researching lines devoted to develop (theory, design, fabrication, characterization and test) integrated devices operating with quantum light

3. Quantum propagation in IO

We describe the main theoretical results on quantum mechanical studies in IO. First of all we establish the technical problem of the quantum propagation together with its special characteristics in IO. Next we present the main results on both the derivation of the Momentum operator in IO (and other operators such as the Spin operator) and the spatial propagation in the optical field-strength space.

3.1 The quantum propagation problem

The quantum analysis in IO is based on spatial propagation of quantum light in wave-guiding structures and therefore in linear and nonlinear inhomogeneous media. Serious problems related to the quantum optical propagation in linear and nonlinear homogeneous media are well-known such as Abram showed in a seminal paper [3] and later Huttner *et al* [4] solved in a very interesting phenomenological way based on the so-called temporal modes. The starting point is that the standard Hamiltonian formulation in a volume V of the space (which, however, is fully suitable for stationary optical modes in a cavity) did not give a consistent description of the quantum light propagation. This inconsistency can be easily suspected from the fact that the energy density is different inside and outside a homogeneous dielectric medium and however the frequency (photon energy) must be a constant. Although such a inconsistency can be partially circumvented by using temporal modes other problems remain such as we will described below. The inconsistency of the Hamiltonian approach is even most important when the medium is a dispersive one, because in this case the optical modes will have different speeds and the connection between time and space is not biunique as in a single mode regime. Finally in IO these problems of physical inconsistency will be still more pronounced due to the modal confinement in integrated waveguides. As it has just been mentioned Huttner *et al* [4], in order to avoid part of the mentioned inconsistencies, presented a quantization procedure for linear and nonlinear homogeneous dielectric media based on both the temporal modes and the Momentum operator $\hat{M}$ which operates as a translation operator and therefore can describe spatial propagation in a right way.

On the other hand we must stress the success of the standard Hamiltonian formulation in a volume V in some well-known quantum propagation problems such as for example in a degenerate parametric amplifier. However it is one of those cases that shows how heuristic is sometimes the science. Indeed, the propagation in a degenerate parametric amplifier for a frequency ω has been commonly studied by starting from the standard Hamiltonian formulation in a volume V of the space, and therefore by solving the corresponding temporal Heisenberg's equations to obtain the temporal absorption and emission operators $\hat{a}(t)$, $\hat{a}^{\dagger}(t)$. The most amazing fact is that the standard Hamiltonian operator $\hat{H}$ has, unless constants, the same form that the operator Momentum $\hat{M}$ and therefore the spatial solutions are obtained under a simple formal change in the temporal solutions, that is, if v is the optical mode speed then $t \to -z/v(\omega) = -zn(\omega)/c$, where $n(\omega)$ is the refractive index in the volume V. Alternatively, we could have made this change in the Heisenberg's equations associated to the Hamiltonian operator and then, in a exclusively formal way, we would obtain the Heisenberg's equations associated to a Momentum operator and besides we would obtain the correct solutions for the spatial operators $\hat{a}(z)$, $\hat{a}^{\dagger}(z)$. Obviously this procedure has nothing to do with a rigorous treatment of spatial quantum propagation and in fact the right result is simply obtained by sheer chance. Indeed, if we had considered the non degenerate parametric amplifier for frequencies ω_1, ω_2 then the formal transition to spatial propagation by a simple change between t and z is no longer possible and several non rigorous (and contradictory) depending-frequency changes (due to material or modal dispersion) would have to be made to derive the correct spatial equations and their solutions. These equations (and solutions) must become, in the semi-classical limit, the classical equations (and solutions). In short, several non rigorous and *ad hoc* changes are required to obtain, starting from the standard Hamiltonian operator, the Momentum operator governing spatial propagation in a non degenerate parametric amplifier. On the other hand, if we consider the case of a degenerate parametric amplifier but without fulfilling the phase matching condition, or even if there was a simple z-inhomogeneity inside the considered volume, then the coupling coefficients, which are obtained by means of the Hamiltonian operator after integrating in the volume V, would be fully different from those ones obtained by the Momentum operator. In fact, the degenerate parametric amplifier without phase matching condition was the problem tackled by Huttner *et al* in their seminal work [4], where both proper coupling coefficients for plane modes (that is, non-guided modes) and correct solution for the absorption and emission operators were obtained.

3.2 Quantum propagation in IO

Let us consider an inhomogeneous medium, such as those ones found in IO, then many more serious differences between Hamiltonian and Momentum operators appear. The most important difference regarding to homogeneous media is that the optical modes are guided, that is, they are confined and besides, as a result, they exhibit longitudinal components (optical vector modes) which can not be ignored. In fact in integrated nano-optics these longitudinal components are still larger than in standard IO. As it will be shown the modal orthogonality relation in an inhomogeneous medium is fully different to that one in a homogenous media, therefore starting from the orthogonality relation for optical modes in IO we were able to derive the right optical field operators and accordingly to calculate the correct Momentum operator [5,18]. Likewise it is easy to see that the standard Hamiltonian formulation fails even in the most simple case in IO corresponding to one guided mode with propagation constant β, unlike the monomode Hamiltonian formulation with plane modes, because the modal longitudinal components are essential to obtain the correct Momentum operator. In other words, the contribution of the longitudinal components into the standard Hamiltonian formulation is positive, however in the Momentum formulation such a contribution is negative, therefore we will clearly obtain different results [5], and with the aggravating circumstance that under the standard Hamiltonian approach the frequency of a photon coming into a inhomogeneous medium would have changed. Therefore it is not obvious to obtain in a single way the Momentum operator starting from a Hamiltonian operator by using *ad hoc* changes. For the same reason, the standard Hamiltonian formulation also fails in a simple linear coupling of two degenerate guided modes such as the ones of a linear synchronous coupler device, that is, with propagation constants $\beta_1=\beta_2=\omega N/c$ where N is the so-called effective index. We would have to redefine *ad hoc* the optical modes and their orthogonality relation to be used in the Hamiltonian operator, and moreover to compensate in a some way the change in the energy of the photon due to the presence of modal longitudinal components. Obviously we could still to obtain the formal solutions by making the change $t\to -zN(\omega)/c$ and by using an unknown modal coupling coefficient κ in the temporal solutions such as Toren and Ben Aryeh [15] proposed heuristically. In short, for this simple case we could derive, starting from the standard Hamiltonian in a volume V, the right Momentum operator after a large number of ad hoc changes which, such as in a non degenerate parametric amplifier, are unjustified and contradictory among themselves. In fact more contradictory changes would have to be made if we had considered an asynchronous coupler $\beta_1\neq\beta_2$ or even modal counter-propagation, such as for example modal coupling in an integrated grating (which implements chromatic filters, optical tuners and so on [1]), because we would need to distinguish formally between negative and positive times and therefore to accept formally temporal directions. In short, it is clear the useless of keeping the standard Hamiltonian formulation for quantum optical propagation in IO, and therefore the need to develop a consistent formulation based on the Momentum operator which is the generator of spatial translations, and therefore of the spatial propagation. On the other hand we must also stress that the proper Hamiltonian operator associated to spatial optical propagation, is given, such as Huttner *et al* have shown [4], by the energy flux, that is, the flux of the Poynting's vector, and not by the energy density in a volume *V*, in a such way that the temporal evolution respects the values of the frequencies involved in the linear and/or nonlinear coupling processes. In the same way it is very important to stress that the Momentum operator corresponds to the momentum flux and not to the momentum density in a volume *V*. In the next subsection we will deep about this issue by presenting the main theoretical results.

3.3 Momentum operator in IO

As it was mentioned above the quantum optical spatial propagation in homogeneous media had been analysed by using the Momentum operator [4] starting from a phenomenological approach, however its study in IO has been only recently considered for 2D modes [5] and for 1D modes [18] where a new phenomenological procedure was proposed to derive in a rigorous way the Momentum operator for any integrated optical device. The main consequence is that with this Momentum operator the optical quantum propagation can be rigorously solved *ab initio*. We must also stress that the quantization procedure is developed in a macroscopic way because the present integrated optical devices are not requiring a microscopic quantum theory. Therefore, by one hand the Hamiltonian formulation can not describe correctly spatial quantum optical propagation but only quantum temporal optical propagation due to interactions in a volume *V*, that is, in cavities, and preferably for spatially stationary problems; on the other hand when guided modes are required, such as it happens in IO, the difference between Hamiltonian and Momentum operators is largely relevant. Consequently a quantization approach in IO based on the Momentum operator is needed. We have proposed a phenomenological quantization approach based on both the orthonormalization property of guided modes on the crosssection of optical guides and the complex vectorial structure (transverse and longitudinal components) of the guided modes [5] in order to derive the Momentum operator. This approach starts by considering an arbitrary *z*-direction of propagation, where *xy* is the plane where the guided modes are confined by a graded refractive index *n*(*x*,*y*), that is, 1D guided modes. Then we have to quantize the linear and/or non linear coupling of forward and backward guided modes and to that end the quantum Momentum operator has to be calculated as a function of the corresponding emission and absorption operators denoted by $\hat{a}_\rho$, $\hat{a}_\rho^\dagger$; subindex ρ is an integer different from zero indicating guided forward modes (and forward operators): ρ>0, and backward modes (and backward operators): ρ<0. Next, the spatial evolution of the operators $\hat{a}_\rho(z)$, $\hat{a}_\rho^\dagger(z)$ are obtained by solving the corresponding spatial Heisenberg's equations [4]. In this context it is important to point out the existence of some works on macroscopic canonical quantization based on non plane modal functions but they focused the attention on the derivation of a quantum Hamiltonian operator and therefore on temporal interactions described by the dynamic of the operators; consequently, the ortho-normalization condition used in the quantization process was previously obtained in a volume *V* of a dielectric medium and therefore with the same shortcomings already commented above.

Previous studies on the quantum propagation in space had proven that the Momentum operator can be calculated by integrating the energy tensor element T^{zz} over the hyper-plane *dx dy cdt* [3,4] which provides a deep meaning to the optical detection of photons on the surface of a photodetector, such as it is explained in [4]. Nevertheless, and to our knowledge, this kind of studies were restricted to simple plane modes, and therefore they can not be rigorously applied to optical devices described by non plane waves. The plane modes do not present longitudinal components like guided modes, and accordingly the corresponding field operators have a very particular expression. We must remark that in several interesting and illustrative studies on non-classical light in guided mode coupling devices [14,16] the corresponding Momentum operators were proposed by transferring, in a straightforward way, the results obtained with plane waves, and therefore without taking into account either vectorial structure or orthonormalization property of guided modes. Likewise, analogies with optical quantum Hamiltonians, describing temporal interaction, were used to construct and propose Momentum operators for coupled mode devices [15]. In our work [5,18] we have extended the pioneering results obtained in [4] to the case of coupled mode quantum propagation in optical guides and thus many of the considerations introduced in [3,4] were justified and the results on linear and nonlinear coupled modes were generalized. In short the primary aim of our work has been to avoid methods based on heuristic arguments or mechanical analogies and thus to obtain *ab initio* the Momentum operators corresponding to coupled modes in waveguiding structures and consequently the Heisenberg's equations governing the spatial coupling between the emission and absorption operators. We present the main steps of the procedure by taking into account that the energy tensor T^{zz} element is a moment flux density and therefore the moment flux [15,19] and accordingly the Momentum operator is given by the expression:

$$\hat{M} = \hat{M}_o^e + \hat{M}_o^m + \frac{1}{2}\iiint_0^T \hat{P}_i \hat{e}_i \, dx\,dy\,cdt \,, \qquad (1)$$

$$\hat{M}_o^e = \frac{1}{2}\iiint_0^T n^2 \left(\epsilon_0\, \hat{\boldsymbol{e}}_\tau \hat{\boldsymbol{e}}_\tau - \epsilon_0\, \hat{\boldsymbol{e}}_z \hat{\boldsymbol{e}}_z\right) dx\,dy\,cdt, \qquad (2)$$

$$\hat{M}_o^m = \frac{1}{2}\iiint_0^T \left(\mu_0 \hat{\boldsymbol{h}}_\tau \hat{\boldsymbol{h}}_\tau - \mu_0 \hat{\boldsymbol{h}}_z \hat{\boldsymbol{h}}\right) dx\,dy\,cdt \,, \qquad (3)$$

with $i=x\,y,z$ and $\tau=x,y$; $\hat{P}_i$ are the components of the linear and/or nonlinear polarization (perturbation) operator of the medium which couples the modes of a non-perturbated wave-guiding structure. In order to evaluate the above general operator we must introduce the optical field operators in IO. For that purpose we have defined, starting from the Maxwell modal equations in IO, the modal norm $\|\mathbf{E}_\rho\| \equiv \|\mathbf{H}_\rho\|$ for a guided mode on a cross section z of an integrated waveguide [1,5], that is:

$$\|\mathbf{E}_\rho\| = \left\{ 2\,\mathrm{sgn}_\rho \iint \left\{\mathbf{E}_\rho \wedge \mathbf{H}_\rho^*\right\} \mathbf{u}_z \, dx\,dy \right\}^{1/2} , \qquad (4)$$

where the function sgn_ρ is defined to be +1 if $\rho>0$, and -1 if $\rho<0$. Next the modal field operators associated with a spatial multimode regime must be derived. Thus, by assuming, for sake of simplicity a monochromatic regime (one temporal mode), the following monochromatic field operators are obtained [18] (their extension to polycromatic modes is straightforward):

$$\hat{\mathbf{f}} = \sum_\rho \frac{\sqrt{\hbar\omega}}{\sqrt{cT}\,\|\mathbf{F}_\rho\|} \left\{ \hat{a}_\rho \mathbf{F}_\rho e^{-i\omega t} + \hat{a}_\rho^\dagger \mathbf{F}_\rho^* e^{i\omega t} \right\}, \qquad (5)$$

where $\hat{\mathbf{f}} = \hat{\mathbf{e}}, \hat{\mathbf{h}}$, and $\mathbf{F}_\rho = \mathbf{E}_\rho, \mathbf{H}_\rho$, and the factor $\sqrt{cT}$ can be regarded as a normalization condition of the temporal mode, where T is related to the detection time. Now by using equation (1) any Momentum operator can be derived; for example, in the case of non coupled guided modes in a guide with refractive index $n(x,y)$ the Momentum operator must be calculated with $\hat{P}_i$ =0, which is so-called the free quantum Momentum operator $\hat{M}_o$, and it is easy to prove that for forward and backward guided modes, with propagation constants β_ρ, it is obtained, after a long but very straightforward calculation:

$$\hat{M}_o = \hat{M}_o^e + \hat{M}_o^m = \sum_\rho \hbar\, \mathrm{sgn}_\rho\, \beta_\rho \hat{a}_\rho^\dagger \hat{a}_\rho \,, \qquad (6)$$

where the following relation and commutation rule for forward and backward modes must be used: $\hat{a}_{\rho<0} = -\hat{a}_\rho^\dagger$, $[\hat{a}_\rho, \hat{a}_{\rho'}^\dagger]=\mathrm{sgn}_\rho \delta_{\rho\rho'}$, which are obtained starting from the reversal-time symmetry of the Maxwell's equations. We must also stress that the above result contains an important physical content: the function sgn_ρ ensures the positivity of the momentum, which was assumed in the heuristic approachs [15] and now, such as it can be proved, it is a natural consequence of the modal norm (4). Obviously spatial quantum optical propagation will be obtained by solving the Heisenberg's equations for the Momentum operator, that is, $-i\hbar\partial\hat{a}_\rho/\partial z$ = $[\hat{a}_\rho, \hat{M}_o]$. Now for the quantization of the Momentum operator for linear and/or nonlinear coupling of guided vectorial modes the same above steps can be followed. In the case of a linear polarization corresponding to an isotropic and inhomogenous perturbation $\Delta\epsilon$, the polarization operator will be $\hat{P}_i = \Delta\epsilon\, \hat{e}_i$, therefore by taking into account equations (1-5) the Momentum operator under linear coupling, after a long but straightforward calculation, is given by:

$$\hat{M} = \hat{M}_o + \sum_{\rho\le\rho'} \left\{ \kappa_{\rho\rho'} \hat{a}_\rho^\dagger \hat{a}_{\rho'} + \mathrm{h.c.} \right\}, \qquad (7)$$

where $\kappa_{\rho\rho}$ are the self-coupling coefficients and $\kappa_{\rho\rho'}$ are the cross-coupling coefficients whose expression can be found in [18]. Likewise nonlinear Momentum operators in IO can be derived in this way, in particular a nonlinear coupler where a second-harmonic modal process is produced in the mode of a waveguide coupled to a linear mode in another guide has been obtained as an example [18]. In this way heuristic operators previously used in IO for this nonlinear coupler [16] have been justified. Obviously the Momentum operator of any other nonlinear coupler can be also obtained.

3.4 Other operators in IO

Recently we have also shown that the approach followed for obtaining the Momentum operator can be also applied to the derivation of other operators for quantum integrated optics [19], that is, the Momentum operator is really based on a conservation law which allows to connect the temporal change of a physical magnitude in a volume with the flux of this magnitude through the surface enclosing that volume and therefore the spatial propagation is described by the new flux density function. Indeed, let us consider a total field $\{\boldsymbol{e},\boldsymbol{h}\}$, that is, a field which can be also written as a linear combination of ortho-normalized vector modes $\{\boldsymbol{e}_\rho,\boldsymbol{h}_\rho\}$, and moreover let us consider the expression of an arbitrary physical magnitude depending on the total field and (although not always) on the electric permittivity, that is, $G(\boldsymbol{e},\boldsymbol{h},\epsilon)$, such as for example the energy density u, the longitudinal component of the momentum density p_z or the longitudinal component of the spin density S_z. It must be stressed that we only considered the longitudinal z-component of the vector magnitudes because in integrated photonic guides the energy is only propagated along this direction and therefore the rest of components do not have any physical relevance. Now, the general expression for the conservation law in a volume of any of these density magnitudes requires a flux density $\boldsymbol{J}$ fulfilling a local conservation law, that is: $\partial G(\boldsymbol{e},\boldsymbol{h},\epsilon)/\partial t+\nabla \boldsymbol{J}=0$; from this equation, by integrating in a volume, it is obtained the z-component of the flux density J_z and accordingly the associated optical quantum operator can be derived from the following equation [19]:

$$\hat{O} = \iint \hat{\boldsymbol{J}}_z\,(\hat{\boldsymbol{e}},\hat{\boldsymbol{h}},\epsilon)\,dxdydt. \qquad (8)$$

This technique has been used to obtain the expression of the Hamiltonian operator, Momentum operator and Spin operator in IO [19], all of them as flux operators such as it is required for the spatial propagation problem. It is worth stressing that this approach could be also applied to quantum electron optics, in particular to electron optics waveguides [20] whose interest has also grown in the last years and whose parallelism with integrated photonic devices is very high.

3.5 Quantum optical propagators

The quantum propagation analysis has been firstly described by using the spatial Fock states and to this end the Heisenberg's equations for the Momentum operator along with the operators creation and annihilation associated to the coupled guided modes have to be solved. Alternatively, the quantum propagation analysis can be made in the optical field-strength multimode space, that is, by using the optical-field states, where the probability amplitude distributions in the Schrödinger's image are propagated along a spatial direction. Both kind of analysis can be applied to different modal coupling problems between optical guides, which, as it was already commented, form the basis of many linear and non linear integrated photonic devices, such as: integrated couplers, integrated gratings, integrated junctions, integrated interferometers and so on. All these coupled modes optical devices provide very interesting dynamic and topological properties, therefore a detailed analysis of the quantum propagation will allow to know the optical-quantum potentiality of these devices. First of all we were interested on the resolution of the Heisenberg's equations in the linear case [21], where we must stress that such equations described simultaneously both modal co-propagation and counter-propagation, that is, coupling between forward and backward modes, which in turn has no counterpart in temporal evolution. This is one of the most relevant results obtained by using a quantum optical propagation theory based on the Momentum operator because a Hamiltonian formulation clearly fails for coupled modes counter-propagation. Likewise the spatial propagation in nonlinear integrated devices was studied, in particular a coupler formed by a linear guide and a nonlinear guide where a second harmonic generation is produced. The analysis of this nonlinear coupler shows interesting properties about the spatial propagation of the quantum noise [22].

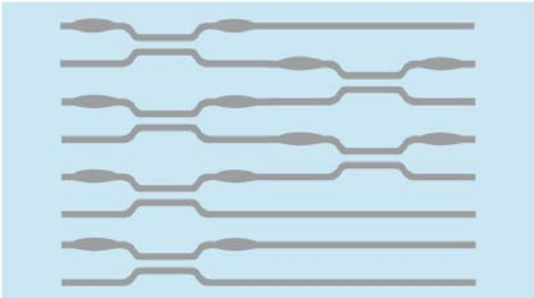

Fig. 2: Directional couplers in cascade.

At present, we are focused on the study of optical quantum spatial propagation in integrated linear and nonlinear directional couplers in cascade such as that one showed in Fig. 2, with applications to quantum processing, and quantum computation in particular, quantum sensing and so on. Moreover we are also interested in the description of spatial forward propagation in the Schrödinger's image and on the optical field-strength multimode space. This interest is based on the appealing and powerful homodyne and heterodyne detection techniques that provide the measurements of the optical-field strength probability distributions of the quantum states. We briefly present the starting point of this description based in turn on the use of the Feynman formalism. We start by establishing the general expression of the spatial optical quantum propagators. The optical field-strength space is defined by means of the optical-field states $|E_\rho\rangle$, which are eigenstates of the optical-field operator $\hat{E}_\rho=(\hat{a}_\rho+\hat{a}_\rho^\dagger)/2$ with eigenvalues ε_ρ, therefore we can represent any N-dimensional quantum multimode state $|L\rangle$ by using the multimode optical field states $|E_1...E_N\rangle$, that is, by means of a complex quantum amplitude on the optical field-strength space and consequently by the corresponding probability distribution $|L\rangle \equiv A(\varepsilon_1,...\varepsilon_N)$. On the other hand, by following the path-integral formalism proposed by Feynman, we can calculate the spatial propagation of the complex quantum amplitude by the following expression:

$$A(\boldsymbol{\varepsilon}_b;z)=\int...\int K(\boldsymbol{\varepsilon}_b,\boldsymbol{\varepsilon}_a;z)A(\boldsymbol{\varepsilon}_a;0)d\boldsymbol{\varepsilon}_a\,, \qquad (9)$$

with $\boldsymbol{\varepsilon}_b=(\varepsilon_{1b},...,\varepsilon_{Nb})$, $\boldsymbol{\varepsilon}_a=(\varepsilon_{1a},...,\varepsilon_{Na})$ and where $K(\boldsymbol{\varepsilon}_a,\boldsymbol{\varepsilon}_a;z)$ is the quantum optical propagator on the multimode optical field-strength space, and $A(\boldsymbol{\varepsilon}_a)$, $A(\boldsymbol{\varepsilon}_b;z)$ are the complex quantum amplitudes at the points z=0 and z. The quantum optical propagator can be expressed as a path-integral on the multimode optical field-strength space ε_1, ..., ε_N, that is,

$$K(\boldsymbol{\varepsilon}_b,\boldsymbol{\varepsilon}_a;z)=\int \exp\{-\tfrac{i}{\hbar}S(\boldsymbol{\varepsilon}_b,\boldsymbol{\varepsilon}_a;z)\}D\boldsymbol{\varepsilon}(z)\,, \qquad (10)$$

where $D\boldsymbol{\varepsilon}(z)$ formally indicates that integration is performed over all paths starting at $(\boldsymbol{\varepsilon}_a;0)$ and ending at $(\boldsymbol{\varepsilon}_b;z)$; the action S, which can be related to an abstract optical path length [23] on the optical field strength space, can be written, by following to Feynman, as follows:

$$S(\boldsymbol{\varepsilon}_b,\boldsymbol{\varepsilon}_a;z)=\int_0^z \sum_{\rho=1}^{N} p_\rho \dot{q}_\rho + M(\boldsymbol{p};\boldsymbol{q})dz\,, \qquad (11)$$

where we established $\dot{q}=\partial q/\partial z$, $\boldsymbol{p}=(p_1, ..., p_N)$, $\boldsymbol{q}=(q_1, ..., q_N)$, and where $M(\boldsymbol{p};\boldsymbol{q})$ must be derived starting from Eq. (1). For instance for the simple case of linear coupling this Momentum can be obtained from equation (7). Note that for each variable ρ=1,...,N we have the following relationships: $\hat{q}=c_o(\hat{a}+\hat{a}^\dagger)/2=c_o\hat{E}$ (that is, $q=c_o\varepsilon$), with $c_o=(2\hbar/\beta)^{1/2}$, $\hat{p}=c_o i(\hat{a}-\hat{a}^\dagger)/2$, and moreover $\dot{q}=-\partial M/\partial p$. With these relations we have applied a Legendre transformation to the classical momentum M for spatial propagation in order to find the Lagrangian-type functional $F(q_\rho,\dot{q}_\rho)$ corresponding to the integrand in equation (11), in the same way as for temporal propagation a Legendre transformation is performed between the classical Hamiltonian and the classical Lagrangian functional. We have obtained the optical propagators for the case of a degenerate parametric amplifier inserted in an integrated directional coupler. The study and analysis of spatial propagation of several singular quantum states in this device is in progress.

4. Quantum propagation in IO

After the above discussion it seems natural to analyze the quantum states $|L\rangle$ with complex probability amplitude $A(\boldsymbol{\varepsilon})$ and therefore the probability distribution $P=|A(\boldsymbol{\varepsilon})|^2$ on the optical field-strength space ($\boldsymbol{\varepsilon}$-space). Its main advantage is that it can be measured by homodyne and heterodyne techniques and moreover, as it will be indicated, the concept and results of quantum polarization can be used to characterize the quantum states on the multimode $\varepsilon_1 ... \varepsilon_N$ space. On the other hand, polarization of quantum states has been an area of great research. One of the most recurrent problems has been that the transition from classical to quantum polarization is not an easy subject, that is, by starting from the quantum Stokes parameters, obtained by quantization of the classical Stokes parameters, several paradoxical results are obtained when the semiclassical degree of polarization is calculated [24-26]. Likewise, the standard concept of polarization fails for both quasi-classical states (like coherent states) and states very apart from them, like number states, because the mean value of the optical field operator does not follow a definite curve of polarization on the two-mode optical field-strength plane. It must be highlighted that important results have been carried out about polarization of quantum states from different perspectives. One of them is based on the $SU(2)$ coherent states by introducing a suitable definition of the degree of quantum polarization as a distance measure to an unpolarized distribution and characterizing the quantum polarization states on the Poincarè sphere [24,25]. Another important perspective is based on the fact that unpolarized light can be considered as a quantum state invariant under any $SU(2)$ polarization transformation [26]. On the other hand, by following closely the ideas of Luis and Sánchez-Soto et.al. [24-26], we have developed a formalism of quantum pseudo-polarization on the N-dimensional optical field-strength $\varepsilon_1 ... \varepsilon_N$ space [27], that is, we take advantage of the polarization concept which, in our case, is understood as the degree of confinement of the probability distribution of the optical field-strength (or position quadrature) on particular regions of the N-mode optical field strength space (pseudo-polarization). It was applied, in a preliminary way, to two-mode quantum states (obviously it becomes standard quantum polarization when the photons are excited on polarization modes), which present a great technological potential, such as two-mode single-photon states, two-mode squeezed states and so on. The main advantages of using this formalism are that, apart from the fact that this formalism is made on the plane where classical polarization is described, the probability distribution is well behaved, it can be directly measured by two-mode homodyne detection and moreover a very simple characterization of the quantum state can be made by defining a polarization degree. This formalism can be applied to spatial modes, such as the modes of a N-mode optical waveguide, a N-mode directional coupler and so on in IO.

We illustrate this analysis with the simple case of an elliptical-type polarization for a two-mode single-photon state $|L\rangle=\sum_{i=1}^{2} c_i |2-i, i-1\rangle$ shown in Fig. 3, which for $|c_1|\neq|c_2|$ resembles the standard elliptical polarization because of the high confinement of probability around a region such that the main maxima of probability are along an axis of the $\varepsilon_1\varepsilon_2$ plane and the secondary maxima are along the orthogonal axis, in an analogous way to the major and minor axis of intensity in classical elliptical polarization.

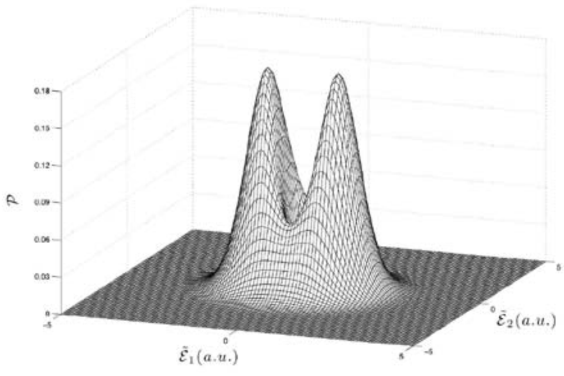

Fig. 3: Two-mode single photon state on the $\varepsilon_1\varepsilon_2$ plane.

This distribution will have an annular shape for circular-type polarization ($|c_1|=|c_2|$), and with two maxima along a straight line for linear-type polarization ($|c_1|\neq|c_2|=0$). On the other hand, as it was mentioned above, to characterize the quantum states a quantum pseudo-polarization degree G can be used (if polarization modes are used then it must be called polarization degree). A general and detailed analysis on this important issue, by taking into account general probability distributions and states, can be found in different specialized references [25,26]. We have used a distance measure between the probability distribution of a quantum state and the probability distribution of a non-polarized state on the $\varepsilon_1 ... \varepsilon_N$ space which can considered as an application of the very general definition given by Luis [24,25] for the quantum polarization degree. Actually we obtain a depolarization degree $\overline{G} = 1 - G$ by calculating the best overlapping between the probability distribution of a Gaussian state (non-polarized state) and the probability distribution of the quantum state $|L\rangle$ to be characterized [27]. We must stress that although an uniform distribution would represent the fully non-polarized quantum state, it would not be physically consistent on the $\varepsilon_1 ... \varepsilon_N$ space because the probability at infinity can not be different from zero. Likewise the results are also fully consistent, thus any quantum state very close to the N-mode vacuum state, such as for example, coherent and chaotic states with a very low photonic excitation, will have a very low quantum pseudo-polarization degree and coherent states with a large mean photon number have a pseudo-polarization degree close to unity, such as it was expected.

5. Conclusions and outlook

We conclude indicating that the main researching work on quantum IO will be the development (that is, theory, design, fabrication, characterization and test) of integrated directional couplers in cascade for different quantum purposes but mainly to implement integrated devices for multimode homodyne and heterodyne detection. We will continue with the study of quantum optical propagation of quantum states in nonlinear coupler devices on the $\boldsymbol{\varepsilon}$-space, as well as the analysis and characterization by quantum pseudo-polarization concepts of the new quantum states generated in these devices.

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