Sección Especial en Óptica Cuántica / Special Section on Quantum Optics

Quantum phases of superradiant matter waves

Fases cuánticas de ondas de materia superradiantes

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ABSTRACT:

Ultracold atoms may be used to observe collective phenomena which are the matter-wave analog of the collective emission of light. Recently, we have studied the emission of atoms initially trapped by an optical lattice into a continuum of free modes. We have shown that this situation is formally equivalent to the emission of light by atoms inside a photonic crystal. Furthermore, we have studied several quantum phases of these models. These include, for example, phases where atoms may be emitted in preferred directions, or phases where emitted atoms form a condensate.

Keywords: Superradiance, Ultracold Atoms, Bose-Einstein Condensation.

RESUMEN:

Los átomos ultrafríos permiten observar fenómenos colectivos que son el análogo en ondas de materia de la emisión colectiva de luz. Recientemente hemos estudiado el problema de emisión de átomos atrapados en una red óptica en el continuo de estados libres, y trazado una analogía de dicho problema con el de la emisión de luz dentro de un cristal fotónico. Además, hemos estudiado distintas fases cuánticas de este sistema, que incluyen, por ejemplo, fases con emisión direccional de átomos, y fases con formación espontánea de un condensado de átomos emitidos.

Palabras clave: Superradiancia, Átomos Ultrafríos, Condensación de Bose-Einstein.

REFERENCES AND LINKS

[1]. I. Bloch, J. Dalibard, W. Zwerger, "Many-body physics with ultracold gases", *Rev. Mod. Phys.* **80**, 885-964 (2008).

[2]. I. de Vega, D. Porras, J. I. Cirac, "Matter-wave emission in optical lattices: Single particle and collective effects", *Phys. Rev. Lett.* **101**, 260404 (2008).

[3]. C. Navarrete–Benlloch, I. de Vega, D. Porras, J. I. Cirac, "Simulating quantum-optical phenomena with cold atoms in optical lattices", arXiv:1010.1730 (2010).

[4]. R. H. Dicke, "Coherence in spontaneous radiation processes", *Phys. Rev.* **93**, 99-110 (1954).

[5]. G. S. Agarwal, "Master-equation approach to spontaneous emission", *Phys. Rev. A* **2**, 2038-2046 (1970).

[6]. D. Porras, J. I. Cirac, "Collective generation of quantum states of light by entangled atoms", *Phys. Rev. A* **78**, 053816 (2008).

[7]. S. John, T. Quang, "Localization of superradiance near a photonic band gap", *Phys. Rev. Lett.* 74, 3419-3422 (1995).

1. Introduction

Cold atoms trapped in optical lattices are ideally suited to study a rich variety of condensed–matter models [1]. In a few recent works [2,3] we have shown that this system also shows a variety of fascinating matter–wave analogs of quantum–optical phenomena. In one way or the other, the physics that we find here are related to the superradiance in the emission of light by atomic ensembles.

The concept of superradiance was introduced by Dicke in 1954 when studying the spontaneous emission of a collection of two–level atoms [4]. He showed that certain collective states where the excitations are distributed symmetrically over the whole sample have enhanced emission rates. Probably the most stunning example is the single–excitation symmetric state (now known as the symmetric Dicke state), which instead of decaying with the single–atom decay rate Γ_0, was shown to decay with $N\Gamma_0$, N being the number of atoms.

It is worth noting that together with atomic ensembles, spontaneous emission of collections of harmonic oscillators were also studied at that time [5]. In the last years, there has been a renewed interest in this phenomenology because of its potential for quantum technologies. Of particular interest to our current work is the analysis performed in [6] of the spontaneous emission by regular arrays of atoms separated a distance d_0 comparable to the wavelength λ of the emitted radiation.

Also of interest for our purposes is the work carried by Sajeev John and collaborators on the collective emission of atoms embedded in photonic band–gap materials [7]. In this kind of materials the density of states of the electromagnetic field is zero for frequencies laying within the gap, what gives rise to the phenomenon of light localization

We have shown that all this phenomenology can be observed in optical lattices with the setup studied in [2,3], which is depicted in Fig. 1. Consider a collection of bosonic atoms with two relevant internal states labeled by a and b (which may correspond to hyperfine ground–state levels). Atoms in state a are trapped by a deep optical lattice in which the localized wavefunction of traps at different lattice sites do not overlap (preventing hopping of atoms between sites), while atoms in state b are not affected by the lattice, and hence behave as free particles. A pair of lasers forming a Raman scheme drive the atoms from the trapped state to the free one, providing an effective interaction between the two types of particles.

In our works, we have considered the situation of having non-interacting bosons in the lattice, as well as hard–core bosons in the collisional blockade regime, where only one or zero atoms can be in a given lattice site. In the first regime, the lattice consists of a collection of harmonic oscillators placed at the nodes of the lattice; in the second regime, two–level systems replace the harmonic oscillators, the two levels corresponding to the absence or presence of an atom in the lattice site.

This system is therefore the cold–atom analog of the quantum–optical systems considered above, with the difference that the radiated particles are massive, and hence have a different dispersion relation than that of photons in vacuum. Moreover, we have shown that the field of free atoms can be characterized by a dispersion relation which is similar to the one obtained for photons within a photonic band gap material [7,2,3]. It is then to be expected, that this system will show the same kind of phenomenology as its quantum–optical counterpart.

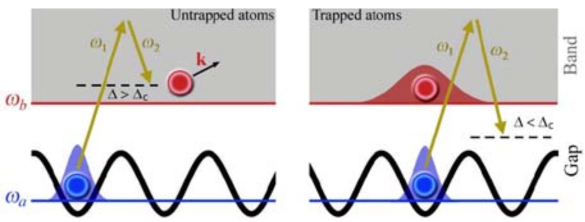

Fig. 1. Scheme for the study of matter–wave superradiance. Atoms are trapped initially by an optical lattice. They are coupled to free states in the continuum by lasers and may get some linear momentum transfer k. These free atoms also get an extra energy Δ. When this energy is below some threshold value Δ_c they cannot escape and form a bound state with the lattice atoms (see Fig. 2); when it is above this threshold they are released out of the lattice.

2. Description of the system

To find a quite detailed description of our system see [3]. Here we only want to sketch a few convincing arguments and equations which show how the idea works.

Our scheme is summarized in Fig. 1. We consider atoms with two internal states, *a* (trapped) and *b* (untrapped), which may correspond to, for example, hyperfine levels as already commented. Atoms in level *a* are trapped in the nodes of the optical lattice (whose positions we denote by j), whereas atoms in level *b* are not trapped at all and can thus have any momentum k. It may happen that the coupling laser transfers some linear momentum to the emitted atoms.

Within the interaction picture, the Hamiltonian that describes the coupling between the trapped and free atoms is the following

$$\hat{H}_{a-b} = \sum_{\mathrm{j,k}} g_k e^{i\Delta_k t - i(\mathrm{k}-\mathrm{k}_L)\mathrm{r}_j} \hat{a}_{\mathrm{j}} \hat{b}_{\mathrm{k}}^{\dagger} + \mathrm{H.c.}, \qquad (1)$$

with

$$\Delta_k = \frac{\hbar k^2}{2m} - \Delta, \qquad (2)$$

and g_k some coupling parameter not relevant for the discussion to come [2,3]. We focus on the case in which the coupling is induced by a two–photon Raman transition, having relative wave vector and frequency between the lasers $\mathrm{k}_L=\mathrm{k}_1-\mathrm{k}_2$ and $\omega_L=\omega_1-\omega_2$ (see Fig. 1), respectively; Ω is the corresponding two–photon Rabi frequency. The presence of the laser detuning $\Delta=\omega_L-(\omega_b-\omega_a)$ in (2) shows that atoms may get some extra negative or positive energy.

The main idea is that Hamiltonian (1) describes the emission of a bosonic field by localized emitters, pretty much as the light–field is emitted by an atomic sample. The bosonic field dispersion has the form of (2), that is, a quadratic dispersion relation with a gap given by the detuning of the coupling laser with the internal transition, Δ. The comparison with the emission of light by atoms inside a photonic band–gap material is clear now, because the effective dispersion relation given by (2) is, indeed, the one corresponding to light at energies near the edge of a photonic gap.

We still have to choose the conditions for atoms in a trapped by the optical lattice. Throughout our work, we have studied two possibilities:

(i) *Hardcore bosons*. This limit is the one which mostly resembles conditions found in quantum optical systems. It consists of atoms in *a* being strongly interacting. In practice, this means that the interaction energy *U* is much larger than the tunneling amplitude between different sites, *t*. See [1] for a review on ultracold atoms in optical lattices.

(ii) *Non-interacting bosons in a lattice*. By neglecting the interaction between atoms, we may consider emission of free bosons by harmonic oscillators, thus finding in our system an experimental realization of some old ideas first considered in Ref. [5].

3. Atom localization

The first interesting phenomenon appears when one considers a single site instead of the entire lattice, that is, one potential well with trapping frequency ω_0 coupled to the continuum. Here we do not have to worry about whether atoms interact or not, as long as we consider the single atom case.

This problem can be solved exactly in the limit $\Omega = \omega_0 \to 0$, and the result shows that for strong enough couplings, a bound state is formed in which free bosons in state *b* are bound to the single well. The state of the system is given by

$$|\psi(t)\rangle = A(t)|1,\{0\}\rangle + \sum_{\mathrm{k}} B_{\mathrm{k}}(t)|0,1_{\mathrm{k}}\rangle, \qquad (3)$$

where $|1,\{0\}\rangle$ refers to the state with the atom in the well and no free atoms, and $|0,1_{\mathrm{k}}\rangle$ to the state with no trapped atoms and one free atom with momentum k.

A careful analysis shows that the strength of the coupling is quantified by the following single parameter [2,3]:

$$\alpha = (2/\omega_0)^{3/2}\Omega^2. \qquad (4)$$

The main result that can be extracted from the solution is summarized in Fig. 2, where we show the evolution of the population remaining in the well $|A(t)|^2$. We see that the ratio between the detuning Δ and α^2 determines the formation of a bound state, since the atom is not able to decay into the continuum.

The plot in Fig. 2 can be interpreted in the following way. For values $\alpha^2 \ll |\Delta|$, the emission of a free atom out of the well depends on the sign of Δ. That is, if $\Delta<0$ the atom does not have energy to be emitted, whereas $\Delta>0$ corresponds to spontaneous emission of an atom out of the well. The case $|\Delta| \approx \alpha^2$, on the contrary, is a strong-coupling limit that corresponds to the formation of a bound–state. The existence of the bound state implies that the atom is distributed around the well, with some non-zero probability of being either in the well or in the continuum of free bosons (see for example the green line in Fig. 2).

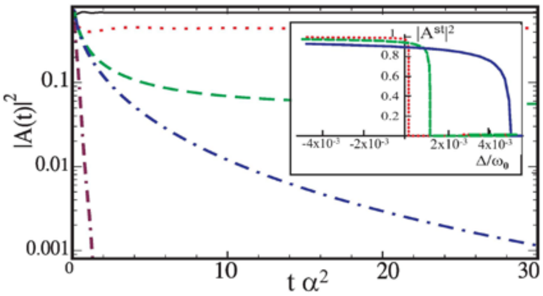

Fig. 2. Evolution of the atomic population $|A(t)|^2$ in logarithmic scale for different detunings in the $\Omega/\omega_0 \to 0$ limit. Solid, dotted, dashed, dot–dashed and dot–dot–dashed lines correspond respectively to Δ/α^2=-8, -1;-0.2, 0.2, 8. Inset: Numerical computation of the steady state population $|A^{st}|^2=|A(\infty)|^2$ without the approximation $\Omega/\omega_0 \to 0$. Solid, dashed and dotted lines correspond, respectively, to Ω/ω_0=0.05, 0.025, 0.01. Note that in this case the phase transition from a bounded to a radiative state is shifted to $\Delta_c = 4\Omega^2/\omega_0$. Taken from [2].

4. Matter-wave superradiance

When going to the many site case we have predicted striking collective phenomena. In particular, the hardcore boson limit turns out to be the analog of the emission of an ensemble of atoms in a photonic band–gap material. We focus in this case, which is a more common situation in quantum optics.

First of all, let us assume that the initial state of the lattice consists of a number of sites M^3 occupied by a single atom in a,

$$|\Psi_0\rangle = \prod_j a_j^\dagger |0\rangle \,. \qquad (5)$$

This apparently simple state is known as Mott insulator. Note that the preparation of such a state is very non-trivial. It results from the fact that having two atoms at the same site is energetically forbidden by the hard–core boson condition. Strong interactions are thus required for $|\Psi_0\rangle$ to be prepared, in much the same way as nonlinearities are required to create Fock states of photons in quantum optics.

The hard–core boson condition also means that the Hilbert space at each site in the lattice consists only of two states, namely, a state with zero or one atom. We can thus replace the boson operators by spin–1/2 operators in Eq. (1),

$$a_j \to \sigma_j \,, \quad a_j^\dagger \to \sigma_j^\dagger \,. \qquad (6)$$

The description of the dynamics of atoms in the lattice may be done by using a master equation. This description assumes the Born–Markov limit of the problem. Consider ρ, the reduced density matrix for atoms trapped in the lattice. ρ satisfies the following evolution equation,

$$\frac{d\rho}{dt} = \sum_{j,l} \Gamma_{j\text{-}l}\sigma_l \rho \sigma_j^\dagger - \Gamma_{j\text{-}l}\sigma_j^\dagger \sigma_l \rho + \text{H.c.} \,. \qquad (7)$$

The couplings $\Gamma_{j\text{-}l}$ –whose exact form can be consulted in [2,3]– describe collective decay rates. j and l are indices that run along the sites of the lattice. Note that in atomic physics one typically finds local couplings of the form $\Gamma_{j\text{-}l} \propto \delta_{j\text{-}l}$. In our work, the couplings are collective, and similar to the couplings one finds when photon reabsorption is included in the spontaneous emission of light by atomic ensembles.

The range of the couplings in Eq. (7) is determined by a typical length,

$$k_0^{-1} = \left(2m\Delta/\hbar\right)^{-1/2} \,. \qquad (8)$$

which plays the same role as the range of, for example, Yukawa interactions.

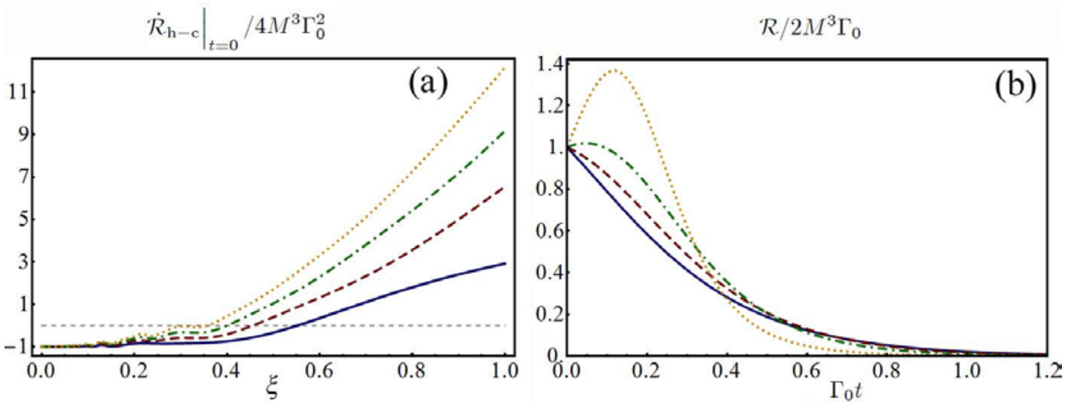

Fig. 3. Collective emission properties for an initial Mott state of hard–core bosons. (a) Time derivative of the rate of emitted atoms R at t=0 as a function of the range of the interactions $\xi=1/k_0d_0$. The values 8 (solid, blue), 27 (dashed, red), 64 (dashed–dotted, green), and 125 (dotted, yellow) are considered for the number of lattice sites M^3. It can be appreciated that there exists a critical value of ξ above which the rate is enhanced at the initial times. (b) Rate as a function of time (M^3=27), with values ξ=0.01 (solid, blue), 0.5 (dashed, red), 1 (dashed–dotted, green), and 10 (dotted, yellow). The maximum of the rate is delayed above some critical ξ value (superradiant pulse). Taken from [3].

It is clear that whenever k_0^{-1} is comparable or much larger than the distance between atoms, say d_0, collective effects will be important. In Fig. 3 we show a few numerical results in the case k_L=0, where some effects related to superradiance appear: enhancement of the spontaneous emission rate and observation of the superradiant pulse, for example.

Operating the system in other regimes, we showed in that it is possible to observe another kind of collective phenomena. For example, when instead of the hard–core limit, the non-interacting limit is considered for the atoms in the lattice, superradiant phenomena associated to harmonic oscillator ensembles [5] is obtained [3]. On the other hand, if the Raman lasers are tuned such that $k_L=k_0$, it is possible to obtain a superradiant atomic beam showing collimated emission within the direction of k_L [2,3], a phenomenon predicted for the emission of light by atomic ensembles in [6].

5. Spontaneous symmetry breaking and atom condensation

A very different phenomenology is observed for couplings strong enough as to make the Markovian approximation invalid. In this limit, we found a situation that, on one hand, is similar to the Markovian regime as atomic bound states can be obtained, but, on the other, is different because the free atoms form a condensate phase.

The study of this limit is particularly difficult, since we find here a true many–body problem. The simplest approximation consists of a mean–field theory in which the state of the system is approximated by a separable state, that is, quantum correlations are neglected. This is indeed a common approximation in many problems of quantum optics like, for example, the theory of the laser, although we face here a situation where we have not only many emitters but also many field modes.

Defining the lattice population and coherence as

$$z=\sum_j \langle \sigma_j^3 \rangle / M^3 \text{, and } y=\sum_j \langle \sigma_j \rangle / M^3 , \quad (9)$$

respectively, it is possible to show that under certain conditions (basically, when the range of the interactions exceeds the length of the lattice, so that all the sites interact with each other with similar strength) the mean field equations can be written as

$$\frac{dz}{dt}=2\int_0^t d\tau \, \mathrm{Re}\{G_{coll}(t-\tau)y^*(t)y(\tau)\}, \quad (10a)$$

$$\frac{dz}{dt}=2\int_0^t d\tau \, G_{coll}(t-\tau)z(t)y(\tau), \quad (10b)$$

where $G_{coll}(\tau)$ is a collective correlation function which gathers the with information of the free modes, see [2].

The non-linear, non-Markovian character of these equations prevent their analytical treatment. However, it is evident that these possess a symmetry: they are invariant under phase transformations of the polarization y. On the other hand, we show in Fig. 4 that there exists a regime in which the non-polarized solution $y=0$ is not stable; any small perturbation leads to a spontaneous build–up of a polarization $y\neq 0$, and hence to a breaking of the phase symmetry. This spontaneous symmetry breaking is a signature of the long–range coherence of the atomic field, and hence of a condensate phase of the free atoms, which in turn is the matter–wave analog of the lasing phase for light.

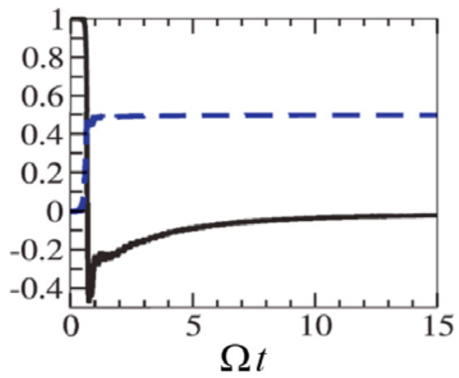

Fig. 4. Spontaneous symmetry breaking and non–zero steady state population. Solid and dashed curves represent, respectively, the population $z(t)$ and the absolute value of the coherence $y(t)$ of a 1D lattice with 100 sites, evolving according to $\Delta=0$ and $\omega_0=50$. Taken from [2].

6. Outlook

The ideas reviewed in this article open many possibilities for studying genuine collective matter-wave phenomena as well as quantum–optical phenomena with optical lattices. Future theoretical work will involve the description of the spontaneous symmetry breaking, and possible applications of our scheme in the generation of entangled atoms and condensates. In particular, to go beyond mean-field theory could improve our understanding of the matter-wave lasing phase, and could help us to think of possible applications of this setup in the engineering of quantum phases of matter.

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