Modelling the dynamics of multimode lasers

Modelado dinámico de láseres multimodo

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ABSTRACT:

In this manuscript we review the task of modelling the dynamics of multimode lasers that we have recently developed. The modelling comprises two different aspects: on one hand, the description of the intracavity field as a superposition of two counter-propagating travelling waves whose amplitudes evolve slowly in time and space; on the other, modeling the optical response of the active medium in such a way that we can correctly describe in time domain its dependence with pumping and frequency. Such a model allows to study the dynamics of multimode lasers in a unified way for a variety of configurations.

Keywords: Laser, Laser Modes, Stability, Dynamics, Mode-Locking.

REFERENCES AND LINKS

1. Introduction

Most laser resonators possess multiple cavity modes which can be excited and forced to lase under appropriate conditions. The material gain and the cavity loss spectrum select which of these longitudinal modes might lase, the degree of competition among them, and thus the dynamical evolution of the system. From the material point of view it is generally acknowledged that inhomogeneously broadened gain media favor the development of multimode action, and although homogeneously broadened lasers (HBL) are widely considered as single mode devices whose dynamics are defined by the cavity mode that lies nearest to the center of the gain line [1-3], they may also develop multimode action [4,5].

From the cavity point of view, most laser resonators are designed in order to sustain a single transverse mode, but many longitudinal modes are still allowed. When the gain difference among longitudinal modes is large — i.e., when the mode spacing is larger than the width of the material gain or when the cavity...
losses strongly favor one mode in front of the others—the dynamics involves only the amplitude of the mode lying the closest to the net gain peak. In this limit, the dynamical evolution of the system can be properly described with the single-mode Maxwell-Bloch equations for the modal amplitude and the corresponding material variables (polarization and population inversion or carrier density in semiconductor lasers). For such a low-dimensional dynamical system, the stability of the solutions and their bifurcation sequences can be thoroughly analyzed as a function of parameters, which has led to a deep understanding of their dynamics [6].

In the general case, a modal decomposition of the intracavity field can be performed, leading to a multimode Maxwell-Bloch model of the system. In this approach, the spectral variations of the material response and the different mechanisms of mode coupling and competition have to be explicitly accounted for, which is usually accomplished with a $\chi^3$ description of the induced polarization. Such models provide an intuitive picture of the mechanisms underlying mode dynamics, and they have allowed to successfully fit the emission spectra under different operation conditions, hereby permitting to determine several important device parameters [7,8]. Nevertheless, one of the main difficulties encountered in this type of models is the large number of parameters required to describe the different nonlinear mechanisms of modal coupling—such as carrier density pulsations, spatial and spectral hole burning, and carrier heating. Another important aspect is that cavity losses lead to spatially varying modal profiles that induce inhomogeneities in the population inversion or carrier density, which generate additional mode coupling terms that are difficult to compute. Finally, one has to determine a priori which modes have to be accounted for in the modelling: increasing the number of modes allows in principle to improve the dynamical description of the system, but at the same time more mode-coupling parameters have to be determined.

Another possibility is to describe the dynamics of the device by directly considering the spatio-temporal distribution of the electromagnetic waves inside the laser cavity. This approach allows to treat on equal footings any cavity configuration by providing the appropriate boundary conditions for the fields. Since no modal decomposition is performed, the number of modes is not limited a priori, and an appropriate time-domain description of the interaction of the optical field with the active medium allows, at least in principle, to include all potential mechanisms of modal coupling with a much reduced parameter set as compared to modal expansions. For these reasons, travelling-wave models (TWM) were developed long ago for solid-state and gas lasers [9], in order to describe multimode dynamics such as modal switching and hopping, or passive and active mode-locking. However, TWM critically rely on having an appropriate time-domain description of the interaction of the optical field with the active medium. Analysis of the TWM for unidirectional ring lasers have shown that HBL can also develop multimode instabilities [4,5] specially when the gain line is not resonant with a cavity mode [10], and experimental evidence exists for self-pulsing at high powers [11], for bichromatic emission [12,13] and for hysteresis among coexisting states as the cavity length is scanned [10,14]. In bidirectional semiconductor ring lasers it has been recently reported [8] that the emission wavelength can be selected by optical injection among that of several modes. Finally, in other systems, the so-called mode-partition noise and mode-hopping [15] arise either dynamically [16,17] or from parameter fluctuations or spontaneous emission noise [18,19].

Such a rich dynamical behavior deserves a deeper understanding, but its study is hindered by the fact that, regardless of the description adopted, the problem involves a large number of degrees of freedom and system parameters. In addition, the situation is still more complex for semiconductor lasers, where a time-domain description of the optical response of the material poses additional burden. Our main research line during these last years has been the implementation of a TWM able to include the optical response of semiconductor materials, as well as the development of semi-analytical tools for analyzing the steady-state solutions and their
stability. In this manuscript, we review this activity.

2. TWM of lasers

We refer the reader to [20-22] and references therein for more details on the TWM approach. The quasi-monochromatic intracavity field is decomposed in a forward and backward waves, whose normalized slowly-varying amplitudes $E_{\pm}$ evolve according to

$$\frac{\partial}{\partial t} E_{\pm}(z, t) = \Gamma P_{\pm}(z, t) - a_i E_{\pm}(z, t). \quad (1)$$

In these equations, $a_i$ are the internal losses of the system, $\Gamma$ is the optical confinement factor, and we have scaled time and space to the cavity transit time and to the optical length of the cavity, respectively. $P_{\pm}$ are the projections of the total material polarization at $(z, t)$ onto the forward and backward propagation directions. They are obtained by a coarse graining procedure by averaging the polarization over a few wavelengths [6].

Since in the TWM the field evolution is governed by a PDE, it is possible to treat on equal grounds ring and FP cavities simply by supplying for the appropriate boundary conditions. In the absence of injection, the general boundary conditions read

$$E_+(0, t) = t_+ E_+(1, t) + r_+ E_-(0, t), \quad (2)$$

$$E_-(1, t) = t_- E_-(0, t) + r_- E_+(1, t). \quad (3)$$

where $r_{\pm}$ and $t_{\pm}$ denote the reflectivity and transmissivity of the forward and backward waves, including the optical phase acquired along the cavity. Setting $r_\pm = 0$ leads to a pure ring cavity, while $t_\pm = 0$ yields a Fabry-Perot device. External injection can be also included by adding the appropriate terms to the boundary conditions.

The total carrier density or the population inversion is normalized to its transparency value $N_t$ and decomposed as $D(z, t) = D_0(z, t) + [D_2(z, t)e^{i\omega_0 z} + c. c.]$, where $\omega_0 = (2\pi n_o)/(\lambda)$ is the optical carrier wave-vector. In this decomposition, $D_0(z, t)$ stands for the quasi-homogeneous component and $D_2(z, t)$ is the (weak) grating component arising from standing wave effects in the system, with $D_{-2}(z, t) = D_2^*(z, t)$. Their evolution is given by

$$\frac{\partial}{\partial t} D_0(z, t) = J - R(D_0) - (P_+ E_+^* + P_- E_-^*) \quad (4)$$

$$\frac{\partial}{\partial t} D_{\pm 2}(z, t) = -(R'(D_0) + 4D_0\delta)\partial_{\pm 2} - (P_\pm E_\pm^* + P_{\pm 2} E_{\pm 2}^*), \quad (5)$$

where $J$ is the (normalized) current density injected per unit time. The recombination term includes the usual non radiative, bi-molecular and Auger recombination terms, as such $R(D) = AD + BD^2 + CD^3$, $R'(D) = dR/dD$ and the ambipolar diffusion coefficient is $D$.

Closure of the system requires linking $P_{\pm}$ to the field amplitudes $E_{\pm}$ and carrier densities $D_0$ and $D_\pm$. For a homogeneously broadened two-level medium with polarization dephasing rate, the material polarization at a given point evolves in time according to

$$\frac{1}{\gamma} \partial_t P = -(1 + i\delta)P + gDE, \quad (6)$$

where $\delta$ is the detuning between the atomic resonance and the optical carrier frequency. Decomposition into forward and backward waves leads to

$$\frac{1}{\gamma} \partial_t P_\pm = -(1 + i\delta)P_\pm + g(D_0 E_\pm + D_{\pm 2} E_{\pm 2}). \quad (7)$$

Semiconductor media are conceptually similar to an ensemble of interacting atoms with different transition energies defined by the electronic hand structure [23], but with different occupancy of each state and fast carrier-carrier interactions. A direct approach to the interaction between a semiconductor material and an optical field is provided by the microscopic semiconductor Bloch equations, that considers each individual transition either including many-body effects [24] or neglecting them as in [23]. These approach offers an excellent description of the material properties, but a dynamical description of the lasing process requires dealing with a large number of coupled, two level like systems. This requires intensive numerical computations [25] which impedes the applicability of microscopic theories for parametric studies.

The computational cost can be reduced by using analytical approximations for the frequency-dependent optical response of semiconductor media [26,27]. A Padé approximation can be used to find the time-domain evolution of the polarization including Spectral-Hole-Burning effects [20], which allows
to successfully describe the behavior of single-section lasers and optical amplifiers. However, the accuracy of the Padé fit to the susceptibility decreases either when increasing the optical bandwidth or when dealing with response functions that vary rapidly in frequency domain, like a Saturable Absorber (SA) close to its bandgap. In this case, the sudden passage -in frequency domain— from transparent to absorptive behavior is poorly described by any kind of fit based on polynomials or rational functions. Noteworthy, the two effects of broadband dynamics and rapidly varying spectral features simultaneously occur in passively Mode-Locked Lasers based on intra-cavity SA [28]. In this case, a convolution method can be developed for the linear susceptibility of the medium under the same assumptions as in [26], which leads to

$$P(z,t) = \varepsilon_0 \int_0^\infty dr \chi[D(z,t-r)] E(z,t-r) \equiv \varepsilon_0 \chi(t,D(z,t)) \otimes E(z,t).$$  

Decomposing into forward and backward waves, we have that

$$\varepsilon_0^{-1} P_{\pm}(z,t) = \chi(t,D_0(z,t)) \otimes E_{\pm}(z,t) + \chi_D(t,D_0(z,t)) \otimes [D_{\pm} E_{\mp}](z,t),$$

with

$$\chi(r,D) = \chi_0 e^{-\gamma r} \frac{2e^{-\Gamma r} - 1 - e^{-\gamma br}}{r},$$

and

$$\chi_D(s,D) \equiv \frac{\partial \chi(r,D)}{\partial \sigma} = -2i\gamma e^{-\gamma(1+iD)r}.$$  

In these equations, $\pi \chi_0$ corresponds to the maximum material gain or absorption, while $b$ is the normalized energy span over which absorption can take place. The rapid decay of the convolution kernels renders this approach practical, since typically the memory segment of integration contains only $K=25$ points, thus the numerical bandwidth is $\approx 4\gamma$ which typically corresponds to 40 THz.

This method allows us to study large bandwidth dynamics with weakly saturating fields, as exemplified in the following sections.

3. Multistability in HBL

The first example of application of our model is the study of the recently reported wavelength multistability in HBL [8] and its dependence on cavity configuration, namely, an (imperfect) ring laser and a Fabry-Pérot laser. This work has been carried out in collaboration with Antonio Pérez Serrano, at IFISC [22].

We numerically determine the modes for a given configuration via a shooting method. With an initial guess for the modal frequency and amplitudes $E_\pm(0)$, we solve for the spatial dependence of Eqs. (1), (4), 5) and (9) using standard numerical integration techniques with a spatial step $h$ towards the other end of the cavity, where the propagated values $E_\pm(1)$ must verify the boundary conditions. A multidimensional Newton-Raphson algorithm provides new guesses for the modal amplitudes and frequency, and the process is repeated until one reaches convergence. Then, we build the temporal map $U(h;\cdot)$ [22] that advances the state vector $V$ a time step $h$ while verifying the Courant condition and cancelling numerical dissipation. We then consider all possible perturbations of $V$ hereby finding the matrix $M$ that represents the linear operator governing the time evolution for the perturbations around one given monochromatic solution. We finally compute the $11\times N$ Floquet multipliers $z_i$ of $M$, which determine the eigenvalues as $\lambda_i = h^{-1} \ln(z_i)$. Repeating this procedure for all possible solutions allows us to obtain a general view of the stability of the system by plotting the bifurcation diagrams for all modes. In our case, however, it suffices to examine only half of the diagram because the resonance condition implies symmetry for $\pm m$.

Figure 1 depicts the general bifurcation diagram for both the ring laser (panel a), and an equivalent FP device (panel b), for which the cavity length is halved due to the double-passage in the cavity. For typical parameters, only bidirectional mode $m=0$ is stable in the ring laser just above threshold. At $J \approx 1$, it is destabilized by a pitchfork bifurcation into two symmetrical, almost unidirectional, solutions also corresponding to $m=0$. Over the interval of $J$ studied, solutions with $m=3$ remain always unstable, but solutions $m=1$ and $m=2$ become
stable for high enough $J$, hence the system easily displays multistability once in the almost unidirectional regime. The equivalent FP laser behaves remarkably different from the ring laser regarding multistability (see Fig. 1b). Above threshold, the mode $m=0$ starts lasing stably, but when the pump is increased it quickly becomes unstable through a multimode instability \cite{4,5}. All the other modes are unstable over all the pump interval examined.

Our analysis reveals that the physical reason for such a different behavior of FP and ring lasers regarding multistability is the quite different degree of spatial hole burning in the gain. In the ring laser, this value saturates at a comparatively low value as soon as the pitchfork bifurcation occurs; for FP configurations, instead, the necessarily higher reflectivity of the facets makes it larger than in the equivalent ring, and it increases continuously with the pump level. As a result, the modal self-saturation of the gain in FP (ring) lasers is larger (smaller) than the cross-saturation imposed by stimulated emission. Indeed, when very high diffusion is considered, FP lasers also display multistability among longitudinal modes.

4. Mode-locking in Fabry-Pérot lasers with SA

The second example of application of our model is the study of mode-locking in two-section Fabry-Pérot semiconductor lasers, where one section provides gain and the other saturable absorption. This work has been carried out in collaboration with Prof. Marc Sorel and his group at Glasgow University \cite{21}. We consider a cavity whose roundtrip time is 25 ps, so the repetition rate of our ML device is 40 GHz, discretized with $N=400$ spatial points. We consider typical parameter values for Al quaternary materials, and we use the same model for the optical response of the gain and SA regions \cite{9}, the only difference being that we consider in the SA be half that in the gain region due to the different carrier densities. Consequently $\chi_S = 2\chi_G$ and the SA band-edge is sharper, as shown in Fig. 2.

The typical sequence of bifurcations found upon increasing of the bias current is: steady emission, weak multimode dynamics, stable ML, a bubble of Self-Pulsation if the SA modulation is sufficiently strong and finally a partial degradation of the pulse train for high drive current (ten times threshold).

The model can be used to optimize device design by analyzing the impact of specific parameters. For instance, when we fix the current to 5 times threshold and we scan the SA length (see Fig. 3) we find that for $l$ ranging from 0% up to 1.25%, only weak multimode can be seen and the optical spectrum is essentially monomode. For $l$ between 1.5% and 2%, a
Fig. 2. Gain and absorption in the gain and the SA sections for different values of the carrier density normalized to transparency within each section.

Fig. 3. Long time behavior of the time traces of the field intensity (left) and details of the pulse shape (right). The panels a), b), c) and d) correspond to \( l = 1.5\% \), \( l = 3\% \), \( l = 3.75\% \) and \( l = 6.5\% \), respectively.

transition to stable shallow Harmonic ML is found, with an amplitude modulation less than 50%. For \( l \) ranging from 2% to 3% unstable ML appears, with substantial modulation of the pulse amplitudes which is accompanied by a considerable jitter in the pulse train. For \( l \) ranging from 3% to 5% stable ML exists with very low noise triggered jitter, while for \( l \) ranging from 5.25% up to 7.5% the device enters a Q-Switch instability regime.

The shortest auto-correlation and the broadest spectra are obtained around 3%, although in a weakly stable regime, simply because too short an absorber is not sufficient to trigger ML due to SA bleaching, and too long an absorber increases losses and in turn decreases the power available to modulate the absorber. In this respect, the statement that shortening the absorber shorten the pulse width, albeit true, is valid only when one is comparing the same current density, as it is the case in Fig. 3. One can also observe that the average frequency is red shifting when the saturable absorber length is increased. The longer the section, the bigger the penalty to operate in the blue when the absorption is large. The laser therefore adapts...
its frequency by shifting from the gain peak to
the red in order to compensate for the quantity
of absorption.

Similarly, other design parameters can be
systematically scanned in order to optimize
device performance within operation
restrictions.

4. Conclusions

We have developed a TWM for modelling the
dynamics of multimode lasers. A crucial
ingredient in the TWM is a qualitatively accurate
optical response of the active medium that
correctly describes in time domain its
dependence with pumping and frequency. Such a
model allows to study the dynamics of
multimode lasers in a unified way for a variety of
configurations.