Descripción de la evolución espacial de un haz gaussiano difractado por una red radial mediante la Difracción Geométrica Local

Local Geometrical Diffraction to describe the spatial evolution of gaussian beam diffracted by radial gratings

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RESUMEN:

En este trabajo hemos propuesto una solución numérica basada en un modelo geométrico de difracción local y haces de rayos con distribución gaussiana generados por el método de Monte Carlo, para el estudio de la evolución espacial de un haz gaussiano al difractarse por una red radial. Hemos comparado con modelos numéricos que dan forma a soluciones propuestas en trabajos anteriores. El resultado de esta comparación es una excelente coincidencia entre los modelos numéricos. Aparecen discrepancias cuando situamos la cintura del haz incidente en la región focal de redes radiales con un gran número de línea y radio pequeño. La causa de estas diferencias es que las aberraciones se han tenido en cuenta en el caso de la difracción geométrica local.

Palabras clave: Difracción, Métodos numéricos, red radial.

ABSTRACT:

In this work we have proposed a numerical solution based on local geometrical diffraction model and Gaussian ray-bundles generated by means of the Monte Carlo method, for the study of the Gaussian beams diffracted by a radial grating. We have compared it with numerical models that give shape to the solutions proposed for this problem in previous works. The result of this comparison is an excellent agreement between the numerical models. An exception has been shown in the case of an incident beam with the waist in the focal region of gratings with smaller radius and a large number of lines. The cause of this dissent is that the aberrations have been taken into account by the local geometrical diffraction.

Key words: Diffraction, Numerical methods, radial grating.

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1.- Introduction.

Radial gratings are widely used in some optical systems to measure angular displacement and velocity, and for splitting frequency shifted beams in heterodyne detection systems. Beam diffracted in radial grating become spatially distorted. There is some previous works characterizing the effect of the grating on the diffracted beam[1-3].

In ref. [2], the concept of effective focal length was introduced. It was demonstrated that the grating behaves as a quadrupole lens with focal lengths, $\pm 2\pi R^2 / N\lambda$, where R is the distance from the grating centre to the incident beam centre, N is the number of lines in the grating and λ is the wavelength. The principal meridians of this equivalent lens are orientated at $\pm 45^o$ with respect to the direction defined by the centre of the grating and the incident point. In ref. [3] the authors claim to improve the focal length description by means of slightly variable focal lengths, which depends on the object distance.

In this work we propose a numerical solution that we have named Local Geometrical Diffraction (LGD). This solution uses a ray-bundle to describe Gaussian beams[4-6], and each ray is diffracted individually, assuming that the radial grating locally behaves as a parallel-groove grating. Also we use the ray bundle method together with the generalized ABCD theory to improve the interpretation of the effective focal lengths presented in Ref. [2-3].

2.- Local Geometrical Diffraction (LGD).

The Local Geometrical Diffraction theory is based on two models. First, the Gaussian beam is modelled by a bundled of rays each of them having a Gaussian-distributed pointing error[4-6].

The geometrical model for a spherical wave is a set of rays converging to, or diverging from a focus. Even if we produce a Gaussian distribution of rays in a transversal plane, they fail to adequately model a Gaussian beam whenever their direction vectors points to a well defined focus. By using the Monte Carlo method, a Gaussian distributed pointing error is applied to each ray, so they fail to hit the focus. Instead, they form a finite size spot equal to the beam waist, ω_0. In references [4-6] it is demonstrated that propagation of such bundles through ABCD systems reproduces the propagation characteristics of Gaussian beams (locations and sizes of the waists before and after propagation)[7]. We use bi-dimensional histograms computed at planes parallel to the grating to compute the 1st order statistics of the ray bundles, mainly the first and second moments. Once again, the widths of the ray bundles so obtained coincide with the widths of the beams they represent.

To take into account the diffractive effect of the grating, we use the conical diffraction formula stated by Harvey[8], which allows the computation of the non-paraxial direction cosines of a plane wave diffracted by a linear grating for any incidence direction. This formula is applied to each ray on the bundle, using a period and grating orientation which correspond to the local period and local orientation of the radial grating at the point of incidence of each ray.

Once all the rays in the bundle are diffracted, the spatial evolutions of the principal widths of the diffracted beam are evaluated. We have used the formalism based on the width tensor[9] which is defined in terms of the moments of the intensity distribution. The principal axes of the beam are defined by the eigenvectors of the width tensor, and the widths along these directions are the eigenvalues.

3.- Models.

We compare the LGD approach with three models that similarly predicts the distortion of beam diffracted by a radial grating. We will call p and q to the distances from the waist of the incident beam to the grating and from the grating to the waist of the converging section of the diffracted beam. We will test the different models by comparing of the $q(p)$ curves.

First we use the interpretation about the focalization behavior of the diffracted beam that is made in ref. [2]. There the authors proposed that the

radial grating approximately behaves as an optical quadrupole, an astigmatic lens in which the main powers are identical but with different signs. This astigmatic lens is centered at the incidence point on the grating, and its main meridians are rotated 45º from radial direction. Then we use the standard Gaussian beam theory along with the thin lens formula to compute $q(p)$[10] in each orthogonal direction. The one dimensional expression is,

$$q(p) = \frac{fp(p-f)+z_R^2}{(p-f)^2+z_R^2} \qquad (1)$$

where f is the focal length and z_R is the Rayleigh range of the input beam.

There are two approaches to compute the focal lengths of the quadrupole that produces the same beam shaping than the radial grating. In the first one[2] (that we will call M1), these equivalent focal lengths were computed by means of the Fresnel diffraction integrals and the stationary phase approximation. The resulting equivalent focal lengths are

$$f_{X,Y} = \pm\frac{\varepsilon R^2}{\lambda}, \qquad (2)$$

where ε is the angular period, R is the incident radius, and λ is the wave length. In reference 3 a different approach was taken. Koch et al employed a different approach. They used LGD and tried to find the conditions under which the rays went into a focus. After some approximations, they got a slightly different expression for the equivalent focal lengths,

$$f_{X,Y} = \frac{2p}{\left[\pm\left(1+\frac{4p^2}{R^2\tan^2\theta}\right)^{1/2}-1\right]\tan^2\theta} \qquad (3)$$

where p is the distance between the plane of the beam waist and the plane of the radial grating, R is the incident radius, and θ is the diffraction angle. Indeed these are not truly focal lengths, as they depend on the object distance, p. We will call the model using these variable focal lengths M2.

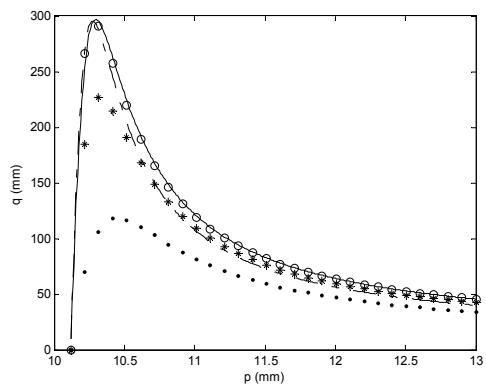
In the third approach, we use the ray bundle to model the incident Gaussian beam. Instead of using LGD to deviate each ray upon diffraction, we substitute the grating by a quadrupole lens centred with the grating (and not with the incidence point as in the previous models). We assume this lens has constant focal lengths as given by [2]. Then, each ray in the bundle is deviated according to the standard ABCD law. We will reference to this model as M3.

4.- Results.

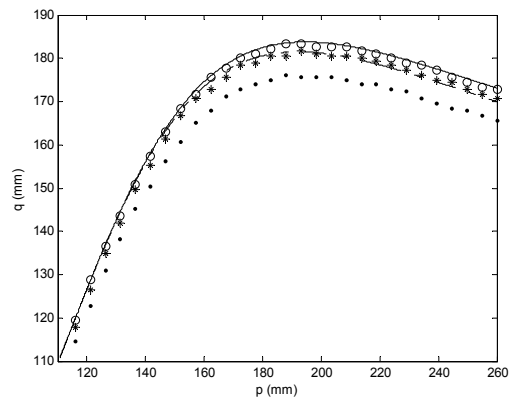
In our calculations we have used two combinations grating/beam with the following characteristics, GB1 (R = 5 mm, N = 24532 lines and $\omega_o = 6\cdot10^{-3}$ mm) and GB2 (R = 22.4 mm, N = 45000 lines and ω_o = 0.13 mm). We use the term *radiality* to qualitatively describe the variation of the local period on the grating across the region irradiated by the incident beam. Two parameters are needed to quantify the radiality. We have chosen the linear period in the center of the beam, $\overline{d}$, and the maximum difference in curvature over the circle of radius equal to the width of the beam spot on the grating, ω. The analytical expressions for these parameters are

$$\overline{d} = \varepsilon R$$
$$\Delta\kappa = \frac{2\omega}{R^2+\omega^2}. \qquad (4)$$

In the case GB1 radiality is large, whereas in case GB2 radiality is fairly low. For the sake of clarity we have shown the curve *q(p)* just for one principal direction.



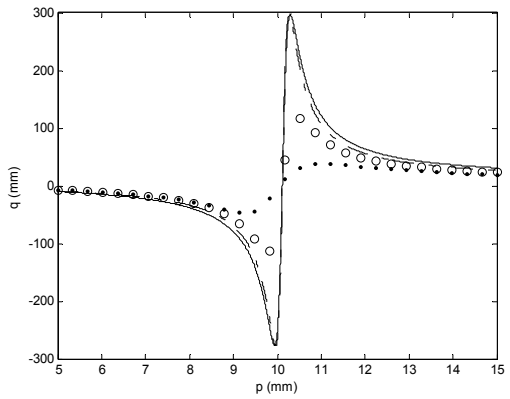
GB1



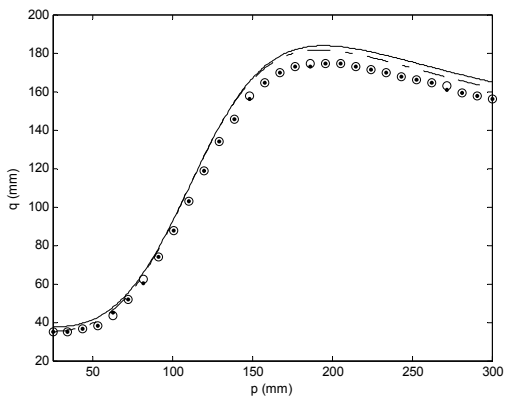
GB2

Figure 1. Effect of put off-centre for two different cases. Models M1 (solid), M2 (dashed), M3 falling on R, R/2 and 0 (points, asterisks and circles, respectively).

The figures shown above confirm a gradual variation of the focusing behaviour as the incident point goes away of the centre. This variation is emphasized when the incident waist is in the neighbourhood of the focal length.



GB1



GB2

Figure 2. Comparison between models. Models M1 (solid), M2 (dashed), LGD (points), and M3 (circles)

The second figure shows the good agreement between the LDG and model M3 in the low radiality case GB2. For the case GB1, there are large disagreements between the models, mainly in the neighbourhood of the focal regions (approximately between maxima and minima of the curve). because of the model LDG takes into count the aberrations of the grating and, M3 involve a perfect quadrupole, without aberrations.

Models M1 and M2 predicts very similar waist position. According to LGD, the maximum and minimum values of q are far smaller. We think this difference is due to the fact that LGD takes into account the large aberrations that are produced in high radiality gratings, whereas the other models are applied to quadrupole lenses without aberrations. M3 gives an intermediate value between LGD and models M1 and M2.

5.- Conclusions.

It has been shown that the numerical model LGD is in a good agreement with the previous numerical solutions found in the literature in the regions faraway from the focal length. Also, we have given a new interpretation of the quadrupole representation by means of the ABCD formalism.

The LGD could be applied to model any diffractive periodic element. The key of this method is to transform the binary approximation of periodic element to a field of slopes.

The same way as other numerical methods, the LGD has its application range for an arbitrary geometry grating. This will come defined by the number of lines and the geometrical properties of the slopes field. To find these conditions is the way that we propose to continue this work.

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