ABSTRACT:
A generalized color-difference space was introduced based on theories of multi-stage color vision, for example, the zone theory, and extended by integrating line elements. This space had the following features: chromatic adaptation, a linear transformation from CIE tristimulus values to cone fundamentals, a nonlinear compression stage, a second linear transformation to opponent signals, and line-element integration that was a function of chroma. Variants of this generalized color-difference space were derived using the RIT-DuPont and Qiao et al. datasets of equal perceived color differences. These spaces had improved performance compared with CIELAB and resulted in a Euclidean distance metric with statistically equivalent or superior performance to the current CIE recommended color-difference formula, CIEDE2000.

Key words: Color-differences, Euclidean color space, Color-difference, Formulae, CIEDE2000.

REFERENCES AND LINKS
1. Introduction

Two stimuli, viewed under identical conditions, match for a specific standard observer when their tristimulus values are equal. This defines basic colorimetry and is the basis for numerical color specification. When their tristimulus values are unequal, the match may not persist, depending on the magnitude of dissimilarity. Relating numerical differences to perceived color differences is one of the challenges of advanced colorimetry. This challenge continues into the present.

There have been numerous approaches to meeting the color-difference challenge, well summarized by Kuehi [1]. Looking back, two pioneers stand out, Müller and Adams. Müller, as a pupil of Hering, recognized that both the Young-Helmholtz trichromatic and Hering opponent theories were necessary to describe color processing, resulting in a multi-stage color vision model. Adams also developed a multi-stage color vision model that was the basis for CIELAB. A key distinction between the two approaches was the inclusion of a nonlinear transformation in Adam’s model. In a conversation with Richard S. Hunter about his color space, I expected him to cite Adams as his inspiration; instead it was Müller. I refer readers to Kuehi [1] and Judd [2] for descriptions of these pioneers.

The Adam’s color space can be generalized, shown in Eq. (1) beginning with CIE tristimulus values:

\[
\begin{pmatrix}
L \\
M \\
S
\end{pmatrix} = \mathbf{M}_{\text{cone}} \begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix}, 
\]

(1)

\[
\begin{pmatrix}
L' \\
M' \\
S'
\end{pmatrix} = \begin{pmatrix}
f(L) \\
f(M) \\
f(S)
\end{pmatrix},
\]

(2)

and

\[
\begin{pmatrix}
W \\
R \\
Y \\
B
\end{pmatrix} \rightarrow \begin{pmatrix}
L' \\
M' \\
S'
\end{pmatrix} = \mathbf{M}_{\text{opponency}} \begin{pmatrix}
L' \\
M' \\
S'
\end{pmatrix}. 
\]

(3)

The first matrix, Eq. (1), transforms tristimulus values to cone fundamentals. Equation (2) describes the nonlinear transformation, commonly a compressive function. In Eq. (3), the compressed cone signals are transformed to Hering opponent signals. \(K\) stands for black, terminology common to graphic arts. CIELAB has this form; however, the cone matrix is a diagonal matrix and the opponent matrix is a \((3\times4)\) matrix because of the -16 in the calculation of \(L'\):

\[
\begin{pmatrix}
L' \\
\alpha' \\
\beta'
\end{pmatrix} = \begin{pmatrix}
0 & 116 & 0 & -16 \\
500 & -500 & 0 & 0 \\
200 & -200 & 0 & 1
\end{pmatrix} \begin{pmatrix}
f(X/X_n) \\
f(Y/Y_n) \\
f(Z/Z_n)
\end{pmatrix}
\]

(4)

where
Given CIELAB’s simplicity, and particularly its “cone fundamentals,” XYZ tristimulus values, it is quite surprising that it has any correlation with color differences. The fact that CIELAB and its antecedent ANLAB have been used for many decades as a first-order color-appearance and color-difference space supports the legitimacy of Adam’s color vision theory and that color-order systems are a reasonable dataset for color-space development. On the other hand, the correlation is poor, especially for industrial tolerances, in the range of 0.5 – 5.0 ∆E*ab. The dominant limitation is a dependency on chroma position, first recognized by McDonald when evaluating ANLAB [3], shown in Eq. (6) for CIELAB using the average chroma of the color-difference pair:

\[
\Delta E_{McDonald}^* = \sqrt{\frac{(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2}{1 + 0.022C_{ab}^*}}. \tag{6}
\]

The need for the chroma compression implies the need for a second nonlinearity in a color space predicting industrial color differences. That is, Euclidean geometry is insufficient to model discrimination and that Riemannian geometry is required [1]. Line elements are integrated, a notable example described by Chasseur who used CMC as a basis for a more uniform color space for industrial quality control [4]. A simple example is using CIE94 to create a uniform color-difference space [5], resulting in Eqs. (7 – 9):

\[
\begin{bmatrix}
L^* \\
a^* \\
b^*
\end{bmatrix} = \begin{bmatrix}
L \\
a^*f(C'_{ab}) \\
b^*f(C'_{ab})
\end{bmatrix}, \tag{7}
\]

\[
f(C_{ab}) = \ln(1 + 0.045C_{ab}^*), \tag{8}
\]

and

\[
\Delta E^* = \sqrt{\frac{(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2}{1 + 0.022C_{ab}^*}}, \tag{9}
\]

The E superscript indicates that Euclidean distances, Eq. (9), correlate with perceived color differences and positional functions, e.g. the denominator of Eq. (6), are not required (in modern color-difference formulae, positional functions are referred to as weighting functions, e.g., S_L, S_C, and S_H [6]).

The McDonald compression assumed that the visual response to lightness and chromaticness (hue and chroma) were equal. Soon after McDonald’s observation, position-dependent functions were derived independently for lightness, chroma, and hue. However, when integrating line elements, hue is not integrated because it has polar rather than rectangular coordinates. Furthermore, color-tolerance ellipsoids fitted to visual data have minimal tilt, that is interaction between lightness and chromaticness. Thus, different integration functions are derived for L* than a* and b*. Examples include Chasseur [4], Rohner and Rich [7], DIN99 [8], Thomsen [5], Luo, et al. [9], and Berns and Xue [10].

One aspect of developing a uniform color space for industrial tolerances is the scaling of lightness. The function described by Eq. (8), even if optimized separately for lightness, results in values less than 100 for the white object color stimulus, a common upper limit, e.g., CIELAB. This is remedied by using a function that maintains scaling between 0 and 100, for example, reference [9].

A second aspect concerns chromatic adaptation and the reference white object color stimulus. Historically, color spaces were derived for illuminant C and the 1931 observer, for example, the various color spaces developed by Friele, MacAdam, and Chickering. Using Eq. (1), when \(L=M=S=1.0\), then XYZ defines the reference white. When CIELAB was derived, the adaptation transform was incorporated into the equations, i.e. \(X/X_0\), \(Y/Y_0\), and \(Z/Z_0\), to enable using the space for a range of reference illuminants not too different from daylight. However, for non-daylight illuminants, this correction can decrease correlation. More accurate chromatic-adaptation transformations can be used, for example, the transform embedded in CIECAM02 [11], shown in matrix form in Eqs. (10) – (12):

\[
\begin{bmatrix}
X_{\text{reference}} \\
Y_{\text{reference}} \\
Z_{\text{reference}}
\end{bmatrix} = M_{\text{CAT02}} M_{\text{VK}} M_{\text{CAT02}} \begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}, \tag{10}
\]

where \(M_{\text{CAT02}}\) transforms XYZ to pseudo-cone fundamentals, RGB:

\[
M_{\text{CAT02}} = \begin{bmatrix}
0.7328 & 0.4296 & -0.1624 \\
-0.7036 & 1.6975 & 0.0061 \\
0.0030 & 0.0136 & 0.9834
\end{bmatrix}, \tag{11}
\]

and \(M_{\text{VK}}\) is the von Kries diagonal matrix:

\[
\begin{bmatrix}
0.566 & 0.232 & 0.1 \\
0.679 & 0.3 & 0.021 \\
0.299 & 0.587 & 0.114
\end{bmatrix}
\]
When using the CIECAM02 chromatic adaptation transformation, “RGB” are used instead of “LMS” to denote pseudo-cone fundamentals rather than physiological cone fundamentals.

It is observed that a simple color-difference space can be derived by combining the concepts of Müller and Adams and the logarithmic line-element integration of Chasseur. The purpose of this publication was to derive such spaces and consider whether this approach should be used for formulae development for industrial color control.

2. General color-difference space

First, CIECAM02’s chromatic adaptation transform was used:

\[
\begin{pmatrix}
X_{\text{Illuminant E}} \\
Y_{\text{Illuminant E}} \\
Z_{\text{Illuminant E}}
\end{pmatrix} = M_{\text{CAT02}}^{-1} M_{\text{VK}} M_{\text{CAT02}} \begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix},
\]

where tristimulus values ranged between 0 and 1 following the transformation. Illuminant E was selected because for either CIE standard observer, X=Y=Z=1. Second, a constrained matrix transformed XYZ to LMS:

\[
\begin{pmatrix}
L \\
M \\
S
\end{pmatrix} = \begin{pmatrix}
e_1 & e_2 & e_3 \\
e_4 & e_5 & e_6 \\
e_7 & e_8 & e_9
\end{pmatrix} \begin{pmatrix}
X_{\text{Illuminant E}} \\
Y_{\text{Illuminant E}} \\
Z_{\text{Illuminant E}}
\end{pmatrix},
\]

where \((e_1+e_2+e_3), (e_4+e_5+e_6), \) and \((e_7+e_8+e_9)\) each summed to unity. These row sums were optimization constraints and were required to maintain illuminant E as the reference illuminant. Third, an exponential function was used for the nonlinear stage:

\[
\begin{pmatrix}
L' \\
M' \\
S'
\end{pmatrix} = \begin{pmatrix}
L'^{1/\gamma} \\
M'^{1/\gamma} \\
S'^{1/\gamma}
\end{pmatrix},
\]

where \(\gamma\) defined the exponent; the same exponent was used for all three cone fundamentals. Fairchild [12] showed that this type of function well fitted the visual data leading to the Munsell value scale and eliminated the need for the complex function used in CIELAB, Eq. (5). This concept was also used in RLAB [13,14] and IPT [15]. Fourth, the compressed cone responses were transformed to opponent signals:

\[
\begin{pmatrix}
W' \Leftrightarrow K' \\
R' \Leftrightarrow G' \\
Y' \Leftrightarrow B'
\end{pmatrix} = \begin{pmatrix}
100 & 0 & 0 \\
0 & 100 & 0 \\
0 & 0 & 100
\end{pmatrix} \times \begin{pmatrix}
e_1 & e_2 & e_3 \\
e_4 & e_5 & e_6 \\
e_7 & e_8 & e_9
\end{pmatrix} \text{opponency,}
\]

where \((e_1+e_2+e_3)=1, \ (e_4+e_5+e_6)=0, \) and \((e_7+e_8+e_9)=0\). These row constraints generated an opponent-type system and were also used as optimization constraints. The fifth step was to compress the chromaticness dimensions to compensate for the chroma dependency:

\[
\begin{pmatrix}
L^E \\
a^E \\
b^E
\end{pmatrix} = \begin{pmatrix}
W' \Leftrightarrow K' \\
R' \Leftrightarrow G' \\
Y' \Leftrightarrow B'
\end{pmatrix} f(C),
\]

where

\[
f(C) = \frac{\ln(1+\beta_C C)}{\beta_C C},
\]

and

\[
C = \sqrt{\left(R' \Leftrightarrow G'\right)^2 + \left(Y' \Leftrightarrow B'\right)^2}.
\]

The Lab terminology was used because of its familiarity. The E superscript implies that it is a color space such that Euclidean distances correlate with perceived industrial tolerances. The final color difference was calculated using Eq. (20):

\[
\Delta E^E = \sqrt{\left(\Delta L^E\right)^2 + \left(\Delta a^E\right)^2 + \left(\Delta b^E\right)^2}.
\]

3. Optimizations

In this research, the RIT-DuPont [16] and Qiao et al [17] datasets were combined and used as the visual and colorimetric dataset for optimization. There were 200 color-difference pairs, all having the same visual difference. The tristimulus values for D65 and the 1964 standard observer were used to define the white object-color stimulus \((X_w=94.81, \ Y_w=100.00, \ Z_w=107.32)\). For each optimization, the objective function was minimizing STRESS, a performance metric enabling hypothesis testing [18]. Constrained nonlinear optimization was used, including derivative- and simplex-type algorithms. Several observations were noted during the course of these optimizations. First, convergence was difficult.
because of the large number of model coefficients requiring estimation. Second, there were many solutions having nearly identical STRESS values, these solutions dependent on starting values. The implication was that these models had too many coefficients, the various stages had insufficient independence, and there were many local minima. As a consequence, it took a number of iterations with setting and optimizing the various coefficients until the coefficients were reasonably stable. The number of significant figures was also minimized to aid in interpreting the results. Thus, although these color spaces may not be globally optimal, they met the requirements for the stated research goals.

**Color-Space 1**

The first optimization used $XYZ$ as cone fundamentals, that is, the cone matrix (Eq. (14)) was a diagonal matrix and a $\gamma$ of 2.3, first introduced by Fairchild [12]. The coefficients for the opponency matrix (Eq. (16)) and the chroma compression, $\beta_c$, (Eq. (18)) were estimated. Color-Space 1 would be CIELAB with a more complex opponent process and without assuming that lightness was a function of only luminance factor.

**Color-Space 2**

The second optimization used the Hunt-Pointer-Estevez (HPE) cone fundamental matrix derived for color appearance models including CIECAM02 [11] and a 2.3 $\gamma$. The opponency matrix and chroma compression constant were estimated. This color space has physiologically plausible cone fundamentals rather than tristimulus values.

**Color-Space 3**

The third optimization used a defined $\beta_c$ of 0.04 and 2.3 $\gamma$ and estimated both matrices. It was of interest to evaluate a set of cone fundamentals derived for color differences without considering physiology. The value of 0.04 was selected because it was the optimal value from both Color-Spaces 1 and 2.

**Color-Space 4**

For the last color space, all the coefficients were estimated.

### Table 1

<table>
<thead>
<tr>
<th>Color Space</th>
<th>Cone</th>
<th>$\gamma$</th>
<th>Opponency</th>
<th>$\beta_c$</th>
<th>STRESS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\begin{pmatrix} 1 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 1 \end{pmatrix}$</td>
<td>2.3</td>
<td>$\begin{pmatrix} 0 &amp; 1 &amp; 0 \ 4.5 &amp; -4.1 &amp; -0.4 \ 0 &amp; 1.5 &amp; -1.5 \end{pmatrix}$</td>
<td>0.04</td>
<td>24.9</td>
</tr>
<tr>
<td>2</td>
<td>$\begin{pmatrix} 0.39 &amp; 0.69 &amp; -0.08 \ -0.23 &amp; 1.18 &amp; 0.05 \ 0 &amp; 0 &amp; 1 \end{pmatrix}$ *</td>
<td>2.3</td>
<td>$\begin{pmatrix} 0.65 &amp; 0.35 &amp; 0 \ 8.5 &amp; -9.5 &amp; 1 \ 1.7 &amp; 0 &amp; -1.7 \end{pmatrix}$</td>
<td>0.04</td>
<td>22.2</td>
</tr>
<tr>
<td>3</td>
<td>$\begin{pmatrix} 1.2 &amp; 0.2 &amp; -0.4 \ -0.7 &amp; 1.1 &amp; 0.6 \ -0.6 &amp; 0.9 &amp; 0.7 \end{pmatrix}$</td>
<td>2.3</td>
<td>$\begin{pmatrix} 0.5 &amp; 0.6 &amp; -0.1 \ 2 &amp; -14 &amp; 12 \ 0 &amp; 16 &amp; -16 \end{pmatrix}$</td>
<td>0.04</td>
<td>20.6</td>
</tr>
<tr>
<td>4</td>
<td>$\begin{pmatrix} 0.6 &amp; 0.5 &amp; -0.1 \ -0.3 &amp; 1 &amp; 0.3 \ -0.2 &amp; 0.4 &amp; 0.8 \end{pmatrix}$</td>
<td>3.2</td>
<td>$\begin{pmatrix} 0.5 &amp; 0.6 &amp; -0.1 \ 5 &amp; -9 &amp; 4 \ 0.8 &amp; 1.8 &amp; -2.6 \end{pmatrix}$</td>
<td>0.05</td>
<td>18.4</td>
</tr>
<tr>
<td>Linearized CIELAB</td>
<td>$\begin{pmatrix} 1 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 1 \end{pmatrix}$</td>
<td>2.3</td>
<td>$\begin{pmatrix} 0 &amp; 1 &amp; 0 \ 5 &amp; -5 &amp; 0 \ 0 &amp; 2 &amp; -2 \end{pmatrix}$</td>
<td>0.045</td>
<td>26.6</td>
</tr>
<tr>
<td>Linearized IPT**</td>
<td>$\begin{pmatrix} 0.4 &amp; 0.7 &amp; -0.08 \ -0.22 &amp; 1.15 &amp; 0.06 \ 0 &amp; 0 &amp; 0.9 \end{pmatrix}$</td>
<td>2.3</td>
<td>$\begin{pmatrix} 0.4 &amp; 0.4 &amp; 0.2 \ 6.7 &amp; -7.3 &amp; 0.6 \ 1.2 &amp; 0.5 &amp; -1.7 \end{pmatrix}$</td>
<td>0.037</td>
<td>23.7</td>
</tr>
<tr>
<td>CIEDE2000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>22.6</td>
</tr>
</tbody>
</table>

*Only two significant figures past the decimal are shown. See Eq. (11) for the actual coefficients.

**The matrices of IPT do not meet the constraint requirements imposed or defined for the other color spaces. IPT was not derived with these constraints.
The results are summarized in Table I. Also shown are two optimizations using CIELAB and IPT where only $\beta_c$ was estimated. For CIELAB, a $2.3$ $\gamma$ was used in place of the usual function, Eq. (5). The term “linearized” was used to reflect that Euclidean (linear) distances correlate with perceived color differences.

4. Cone fundamentals and opponent channels spectra

Each color space resulted in a set of cone fundamentals, plotted in Fig. 1. The input was the 1931 CIE standard observer normalized to illuminant E and rescaled so that the peak of $\gamma$ equaled unity. CIELAB and Color-Space 1’s “cone fundamentals” were color-matching functions, by definition having little resemblance to cones. IPT and Color-Space 2 used physiological-based cone fundamentals, showing the characteristic unimodal characteristics. Color-Spaces 3 and 4 resulted in cone fundamentals that did not result in physiological-type sensitivities. In Color-Space 3, L was sharpened (negative sensitivity) while M and S were bimodal and sharpened. Also, the M and S sensitivities had very similar shape. For Color-Space 4, each sensitivity was bimodal to some extent.

The product of the two matrices was used to calculate opponent spectral sensitivities, plotted in Fig. 2. The peak wavelengths of each opponent channel are listed in Table II. The matrix constraints resulted in opponent-type channels for all four optimizations. For all the color spaces, the redness and blueness peak wavelengths were identical, and the greenness peak nearly so, possibly a result of the various optimization constraints, rather than some fundamental property of color vision. Differences occurred in lightness and yellowness. CIELAB and Color-Space 1 resulted in identical peak wavelengths, though different amplitudes, an expected result since they share common cone sensitivities, color-matching functions. Using cone fundamentals, Color-Space 2, resulted in different peak wavelengths for lightness and yellowness, both occurring at longer wavelengths. Color-Space 3’s peak wavelengths were the most dissimilar with its lightness peak at 570 nm and its yellowness peak at 540 nm. Increasing the degree of freedom with an optimized nonlinearity, Color-Space 4, resulted in peaks more similar to the other color spaces. For Color-Spaces 3, 4, and IPT, the lightness channel was slightly bimodal having a secondary peak at short wavelengths. The sensitivities for IPT indicate that it is different than Color-Space 2 and that its matrices were optimized for different properties than physiology, in this case hue linearity.

Fig. 1. Cone fundamentals.
Table II.

Peak wavelengths (nm) of each color space’s opponent channels.

<table>
<thead>
<tr>
<th>Color-Space</th>
<th>Lightness</th>
<th>Redness</th>
<th>Greenness</th>
<th>Yellowness</th>
<th>Blueness</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIELAB</td>
<td>555</td>
<td>610</td>
<td>530</td>
<td>555</td>
<td>445</td>
</tr>
<tr>
<td>Color-Space 1</td>
<td>555</td>
<td>610</td>
<td>530</td>
<td>555</td>
<td>445</td>
</tr>
<tr>
<td>Color-Space 2</td>
<td>565</td>
<td>610</td>
<td>530</td>
<td>575</td>
<td>445</td>
</tr>
<tr>
<td>Color-Space 3</td>
<td>570</td>
<td>610</td>
<td>525</td>
<td>540</td>
<td>445</td>
</tr>
<tr>
<td>Color-Space 4</td>
<td>560</td>
<td>610</td>
<td>530</td>
<td>570</td>
<td>445</td>
</tr>
<tr>
<td>IPT</td>
<td>560</td>
<td>610</td>
<td>530</td>
<td>565</td>
<td>445</td>
</tr>
</tbody>
</table>

5. RIT-DuPont ellipsoids

The RIT-DuPont dataset was used to generate local contours of equal color difference, that is, ellipsoids. Their projections and crosssections are both plotted as a simple way of indicating tilt between the plotted dimensions and the third dimension. When the two contours are coincident, tilt does not occur. As a frame of reference, the ellipsoids in CIELAB are plotted in Fig. 3. There is the characteristic enlarging of the ellipsoids with an increase in chroma and the orientation away for neutrals for the bluish color centers. Also observed is an enlargement for light and dark neutrals. This is associated with a parametric factor where discrimination is improved when the background lightness is similar to the average lightness of the color-difference pair [19] rather than a fundamental limitation of CIELAB. It is important to note that such ellipsoids are a qualitative representation of the color-difference median tolerance (T50) data as the inherent observer uncertainty and directions of the tolerance vectors have a marked affect on the size and orientation [20]. The optimizations used the tolerance data, not ellipsoid data. The amount of tilt was small. The ellipsoids for all the color spaces listed in Table I are plotted in Fig. 4. Linearizing CIELAB by...
compressing in chroma reduced the chroma dependency but had no effect on rotation. Color-Space 1 resulted in a slight improvement, particularly for color centers 15, 16, and 19. Using all the tristimulus values for each chromatic channel improved performance. The ellipsoids were further improved using physiological-based cone fundamentals. The extent of rotation for the blue color centers was reduced. It was greatly reduced in Color-Spaces 3 and 4. Color-Space 4 resulted in the greatest improvement. Linearizing IPT resulted in similar performance to Color-Spaces 1 and 2 and an improvement over linearized CIELAB. Since IPT was designed for good hue linearity, this resulted in blue color centers with improved rotation, though not as improved as Color-Spaces 3 and 4.

7. Discussion
The deficiencies in CIELAB are well known and in part, trace back to Adams not having cone fundamental data. He approximated the visual system with color-matching functions. Color-Space 1 and linearized CIELAB had equal color-difference performance, shown in Table III where F-tests were performed using the STRESS data to evaluate statistical significance. Color-Space 2 was a significant improvement compared with linearized CIELAB; clearly, using physiological cone fundamentals (HPE) is an important consideration. However, this does not alleviate the blue color center ellipsoid rotation. Ebner adjusted HPE primaries for improved hue linearity and as a consequence, improved the blue rotation limitation when deriving IPT. IPT is equivalent to Color-Space 3, except its objective function was hue linearity rather than color-difference uniformity. The nearly equivalent performance (F of 1.324 compared with the critical value of 1.321) is also an indicator that these two different color spaces using different psychophysical methods resulted in similar trends. Color-Space 4 provided further improvement, though not statistically superior compared with Color-Space 3. The difference between these two color spaces was the difference in nonlinearity, γ. The 2.3 corresponded to CIELAB while 3.2 resulted in further compression. This increase in γ was compensating for the lightness dependency parametric factor. This is seen in the neutral ellipsoids having more similar size in Color-Space 4. This is also seen in Fig. 6 for lightness vs. redness/greenness for Color-Spaces 3 and 4. The ellipsoids for Color-Space 4 have a more consistent size. However, compressing all three channels rather than optimizing a lightness weighting function (i.e., S_L) has changed the lightness scale appreciably; the mid-gray color center has high lightness values. This would result in a poor lightness scale. Color-Space 4 also seems worse in predicting constant hue data compared with Color-Space 3, particularly as a function of lightness.

CIEDE2000 is the current recommended color-difference formula [6] and any new color space must be at minimum, statistically equivalent. Color-Space 4 is statistically superior, although it has other shortcomings as discussed above. All of the other Color-Spaces except linearized CIELAB had equivalent performance. This result supports the fact that many of the terms in CIEDE2000 are correcting for limitations in CIELAB. With a better-constructed color space, a simple color-difference formula can be used with equivalent performance to CIEDE2000.
Fig. 4. RIT-DuPont ellipsoids plotted in redness/greenness and yellowness/blueness chromatic coordinates.
Fig. 5. Ebner and Fairchild visual data of constant perceived hue projected onto redness/greenness and yellowness/blueness chromatic planes.
Fig. 6. RIT-DuPont ellipsoids plotted in lightness and redness/greenness planes.

Table III
F values comparing each listed color-difference space and CIEDE2000 color-difference equation. Bold indicates statistically significant difference between the two color spaces: F(0.975, 199, 199), [F_{c1}/F_{c2}] = [0.76, 1.32]. STRESS values for each color space are listed in Table I.

<table>
<thead>
<tr>
<th></th>
<th>Color-Space 1</th>
<th>Color-Space 2</th>
<th>Color-Space 3</th>
<th>Color-Space 4</th>
<th>Linearized CIELAB</th>
<th>Linearized IPT</th>
<th>CIEDE2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Color-Space 1</td>
<td>1.00</td>
<td>1.26</td>
<td>1.46</td>
<td>1.83</td>
<td>0.88</td>
<td>1.10</td>
<td>1.21</td>
</tr>
<tr>
<td>Color-Space 2</td>
<td>0.79</td>
<td>1.00</td>
<td>1.16</td>
<td>1.46</td>
<td>0.70</td>
<td>0.88</td>
<td>0.96</td>
</tr>
<tr>
<td>Color-Space 3</td>
<td>0.68</td>
<td>0.86</td>
<td>1.00</td>
<td>1.25</td>
<td>0.60</td>
<td>0.76</td>
<td>0.83</td>
</tr>
<tr>
<td>Color-Space 4</td>
<td>0.55</td>
<td>0.69</td>
<td>0.80</td>
<td>1.00</td>
<td>0.48</td>
<td>0.60</td>
<td>0.66</td>
</tr>
<tr>
<td>Linearized CIELAB</td>
<td>1.14</td>
<td>1.44</td>
<td>1.67</td>
<td>2.09</td>
<td>1.00</td>
<td>1.26</td>
<td>1.39</td>
</tr>
<tr>
<td>Linearized IPT</td>
<td>0.91</td>
<td>1.14</td>
<td>1.32</td>
<td>1.66</td>
<td>0.79</td>
<td>1.00</td>
<td>1.10</td>
</tr>
<tr>
<td>CIEDE2000</td>
<td>0.82</td>
<td>1.04</td>
<td>1.20</td>
<td>1.51</td>
<td>0.72</td>
<td>0.91</td>
<td>1.00</td>
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</table>

8. Conclusions
The basic concepts of Müller and Adams remain a viable approach to tackling the color-difference problem. In this publication, two representative datasets were used to optimize an integrated line-element color space where Euclidean distances were correlated with perceived color differences. This approach holds promise and can be repeated using other datasets. However, caution should be taken with the model form. For example, the nonlinearity, γ, should not be optimized; instead, an appropriate positional function, S_L, should be included. One may also choose to add other constraints such as ensuring neutral contours of equal color difference are spherical. Color spaces designed as color-difference spaces often are used for other applications. In this case, limitations such as hue linearity, light and chromatic adaptation, and compensating for other color-appearance attributes, should also be considered. A single color space that has applicability for both color appearance and color difference will be a compromise. However, given the high degree of observer uncertainty for both types of data, such a compromise seems reasonable and a step forward.

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